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Multicriteria analysis and EDO-DEA applied to the evaluation of urban real estate
Análise multicritério e EDO-DEA aplicados na avalição de imóveis urbanos
Análisis multicriterio y EDO-DEA aplicado en la valoración de inmuebles urbanos

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Abstract

The modeling of real systems via inference engines have made great strides in engineering, working with the development of decision support systems in order to achieve greater productivity without losing focus on reliability and quality of services provided. The solution of engineering problems, on several occasions, has focused on studies related to the field of optimization, the solutions do not always show trivial. In Engineering Reviews of the use of computer technology has been restricted to the Least Squares Estimators. Adding to this approach will be demonstrated the possibility of using computer systems using methods such as Data Envelopment Analysis under Dual Optics and Multicriteria Decision Analysis. The solutions that emerge in the market make use of optimization techniques linear and / or nonlinear and are usually solved by the application of commercial software packages, where the understanding of the whole process is compromised. In this sense, the main objective of this paper is to present computational solutions developed for the methodologies described above and promote the comparison of results with the commercial software.

Keywords: Optimization; EDO-DEA; Multi-criteria decision analysis.

Resumo

A modelagem de sistemas reais através de mecanismos de inferência tem apresentado grandes avanços na área da engenharia, colaborando com o desenvolvimento de sistemas de apoio a decisão, de forma a se alcançar uma maior produtividade sem perder o foco na confiabilidade e qualidade dos serviços prestados. A solução de problemas da engenharia, em diversas ocasiões, tem foco nos estudos voltados à matéria de Otimização, cujas soluções nem sempre se mostram triviais. Na área de Engenharia de Avaliações, o uso das tecnologias computacionais tem sido restrito aos Estimadores dos Mínimos Quadrados. Acrescentando-se a essa abordagem será demonstrada a possibilidade do uso de sistemas computacionais com a utilização de métodos como a Análise da Envoltória de Dados sob Dupla Ótica e a Análise Multicritério à Decisão. As soluções que se apresentam no mercado fazem uso de técnicas de otimização lineares e/ou não lineares, e normalmente são solucionados com a aplicação de pacotes de softwares comerciais, onde o entendimento de todo o processo fica comprometido. Neste sentido, o principal objetivo desse trabalho será apresentar as soluções computacionais desenvolvidas para as metodologias descritas acima e promover a comparação dos resultados com os softwares comerciais.

Palavras Chaves: Otimização; EDO-DEA; Análise multicritério a decisão.

Resumen

El modelado de sistemas reales a través de motores de inferencia ha dado grandes pasos en la ingeniería, trabajando con el desarrollo de sistemas de soporte a la decisión con el fin de lograr una mayor productividad sin perder el foco en la confiabilidad y calidad de los servicios prestados. La solución de problemas de ingeniería, en varias ocasiones, se ha centrado en estudios relacionados con el campo de la optimización, las soluciones no siempre se muestran triviales. En las revisiones de ingeniería, el uso de la tecnología informática se ha restringido a los estimadores de mínimos cuadrados. Sumado a este enfoque se demostrará la posibilidad de utilizar sistemas informáticos utilizando métodos como el Análisis de Envolvente de Datos bajo Óptica Dual y el Análisis de Decisión Multicriterio. Las soluciones que surgen en el mercado hacen uso de técnicas de optimización lineales y / o no lineales, y generalmente se resuelven mediante la aplicación de paquetes de software comercial, donde se compromete la comprensión de

todo el proceso. En este sentido, el objetivo principal de este trabajo es presentar soluciones computacionales desarrolladas para las metodologías descritas anteriormente y promover la comparación de resultados con el software comercial.
Palabras clave: Mejoramiento; EDO-DEA; Análisis de decisions multicriterio.

Introduction

The representation through mathematical models of systems and phenomena observed in the real world, has taken great strides in recent years, contributing to the development of decision support systems in order to achieve greater productivity without losing focus on reliability and quality of services provided. In Engineering Reviews of the use of computer technology has been further restricted, with the widespread application of Least Squares and Inferential Statistics.

Most often linear or linearization through mathematical transformation models have been used to describe the behavior of the housing market. However, a large part of these processes, regardless of its nature, presents and nonlinear behaviors that lead to complex models requiring analytical and/or numerical solutions, which may not represent the reality under study. When using the transformation of variables, example of Box-Cox transformations, seeking the linearization of the variables, and especially when this practice is applied to the dependent variable, this complexity restricts and hinders knowledge and analysis of the processes themselves.

The process of property assessment involves estimation of various population parameters concerning selected to represent the behavior of the housing market variables. Implicitly, these variables - called independent variables or input - relate to the value of the property - dependent variable or output - non-linear fashion. There is a clear need for new techniques aiming to represent the process of property valuation. The limitation of the least squares estimators using multiple linear regression is focused on mapping a linear approximation of market data. This linear approximation can, in many situations not reflect the market value of the property assessed, especially when the data are highly scattered and intrinsic and extrinsic the most varied characteristics.

In this paper, two methods for joint estimation of the value of urban real estate market are developed: a Data Envelopment Analysis under Dual Optics - EDO-DEA and Multicriteria Decision Analysis. Apart from these methods for estimating the value of the property market, a comparison of results with the least squares estimators, classic method currently used in the evaluation of properties will be presented. Finally, conclusions and future prospects are pointed.

Process of estare rate – Urban Property

A. Real Estate Market

The value of urban real estate market is an important parameter for decision making in many governmental or private agencies: municipalities (development of generic values of plants, collecting real estate taxes, expropriation); Service Union Assets (collection Laudemio, forum) , IRS (taxation of capital gains, the possibility of verification or super transactions valued below involving money laundering); INCRA (paying institution for rural expropriations of land reform); Judiciary (reviews to support judgments) , Financial Agents (guaranteed funding, limit loans, auctions) and private companies (buying and selling, feasibility analysis of projects), among others. The real estate market can be divided into segments, as the market for apartments, houses, shops, offices / run walk, land, and plots (urban or rural), warehouses, parking spaces, etc. Another division is the market for sale or for lease.

The real estate market has a distinct behavior of other markets such as the market for cars, appliances, and other goods of this nature. The main factors that distinguish the properties of other assets is the high life, the uniqueness, its location and spatial fixation and interference of local, state, and federal laws. The service life of the assets of the real estate market may provide difficulties in measuring its value, as this is influenced by the physical characteristics concerning the standard of finishing and repair. Properties with the same actual age may be at different states of conservation, because of building maintenance performed over time and the pattern of their finishes.

Unlike other property where the intrinsic and extrinsic characteristics are not as differentiated's Real estate market are unique. For more coincidental that are the characteristics of certain properties, at least its position or location will be different and there is a property in the real estate market like another. In this sense, on many occasions, determining the value of a property is no easy task and requires the application of scientific knowledge.

The housing market is in a dynamic mechanism, with changes over time and is affected by several factors, whether valorize or devaluating. The simultaneous and disorderly action of many developers, entrepreneurs, builders, and the government itself has the consequence of changing this market, which directly reflects the values by which the properties are offered or traded. In general, it can be seen that the housing market has important components, constantly interacting, and are responsible for the formation of prices for homeowners.

B. Market Value, Price and Cost

The NBR 14.653, Part 1 - General Procedures in Section 3:44, defines market value as: "Market value: Amount by which most likely negotiate voluntarily and knowingly a well, a reference date, under the conditions of the current market."This most likely amount by which it voluntarily and consciously negotiates a good is not necessarily the price at which the good will be traded or offered. The market value is the result of a mathematical and / or statistical modeling process data. These data are obtained by collecting information on the prices at which the properties with similar characteristics are being negotiated or offered.

Therefore, price and value are distinct references. While market value refers to the most probable value of a good, the price reflects the monetary amount by which a particular good is offered or traded. It is extremely common result of a review is different from the price on offer or transaction. What is not expected is that this difference is quite large, in which case there are probably issues to be addressed to justify the value adopted.

The cost of an asset does not reflect the market value because the previous definition of market value, not always the most likely value at which the well will be negotiated to coincide with the cost of production. The market value may be less than, equal to or above the cost of production.

C. Basic approaches in Valuation of Assets

The identification value of a market may well be carried out generally with the use of three different basic approaches:

- income, where the market value of the property is identified from the income it can generate for its economic life;
- the comparison, which is based on prices of similar goods prevailing in the market;
- the cost, which is the basis for calculating direct and indirect costs required for the production of good.

Among the three approaches, the direct comparison is the most suitable and reliable for the identification of market value due to its simplicity and use less subjectivity to get to it. However, evaluations of projects this method can be configured by comparing the price of its shares, the assessments of economic and / or financial, or specific market indicators by comparing the multiplication factors of production (case of gas stations, bakeries, lottery shops, etc..).

D. The Comparative Method of Market Data

The part 2 of ABNT NBR14653 sets that should be prioritized using the comparative method of market data. It impossible to use this method, you can opt for another method that is appropriate for the type of study.

It is emphasized that the decision by the consumption of a particular good, in general, is through the comparative method. The higher the value of the goods to be consumed, the greater should be reviewed for accuracy as well as the “tighter” the economy, more will be needed to effect a transaction based on a valuation report. To measure the market value of an object use, intuitively, to compare this with similar objects and with known values, a procedure called Comparative Process. When we buy any product, such as cars or appliances, first noticed the price requested by several suppliers, then we form a concept about the average price, to finally decide on the purchase, according to our interest, payment terms and financial availability.

When using the Process Comparative, we seek to actually infer a value that is representative for evaluating object, based on other objects that hold similarities between them and the differences that may exist are small or negligible. As knowledge of all objects (the population) available in a particular market is normally inaccessible in its entirety, we made use of samples, whose average values provide estimates of the average value of all the objects that make up the population.

Clearly, the more homogeneous the population investigated, will be more homogeneous sample. Thus, we seek the value of an automobile zero Km, the particular brand, model and year, or a TV set with make and model, it is likely that the samples collected contain prices close to each other and also close to the arithmetic average of all samples collected.

This fact occurs due to the costs of purchasing the products from the manufacturers are similar. Another important issue is the ease of obtaining a representative sample to the market and that greatly favors the obtaining reliable conclusions about the population mean in these cases.

On the other hand, to measure the market value of a property by Method Comparison, the evaluator faces significant difficulties, especially considering that the population is very heterogeneous, also generating heterogeneous samples. The offered products have no brand or sufficiently standardized to make them homogeneous models. Also, do not directly depend on the costs of production, and is often linked to cultural, locational, and socio-economic phenomena, as previously listed.

In both cases, the samples show variation around their arithmetic average. The difference is that in homogeneous samples of processed products such variation is reduced, while the heterogeneous samples, which serve as the basis for measuring the average value of the real estate market, in contrast, usually show a high variation around their arithmetic mean. This fact generates a high degree of uncertainty in the conclusions about the population mean of this product. In reality, any sample taken at random, may contain data with distant values of their arithmetic mean. You could say that the difference between the data and the sample mean is divided into three parts:

- physical differences between the data;

- socio-economic factors;
- randomness in the market.

The portion related to randomness, always present in any market can be defined as an inherent human being when you assign a price to the product you wish to sell or accept the price of a product on the purchase subjectivity. Thus, it can not be measured and is composed of error or noise from a sample taken at random. The portion relating to the physical differences between the data is nil or almost nil in samples of processed products, but extremely important in samples of the housing market, causing great heterogeneity in these samples. These physical differences are due to intrinsic and extrinsic characteristics of properties. So, we can measure these differences, data on real estate market resembled more features possible are collected. Therefore, to use the Comparative Method, is essential to the gathering of this information and, consequently, the existence of a database will be a major factor in the speed and quality of work being performed.

First Approach – EDO-DEA

The DEA has emerged as a tool to evaluate the effectiveness of a combination of production units, differing from traditional statistical approach in that it does not intend to relate a particular unit with the mean or median of the results of comparable units, but with more efficient unit (benchmarking). Rather than being a method focusing on measures of "central tendency", the intention is to be located in the extreme units that make up the "frontier or envelopment" of the data.

The story begins with a dissertation to obtain Edwardo Rhodes Ph.D. under the supervision of WW Cooper published in 1978. The problem addressed aimed at developing a method for comparing the efficiency of public schools, without resorting to the discretion of weights for each variable under study.

The mathematical programming models provide a way to build a border to a particular technology, from an observation of real data, and calculate the distance of the boundary of each of the individual observations. Using this tool for engineering assessments was first proposed by Lyra (2002) and can be summarized as follows:

- the buildings that are part of the sample in the housing market are interpreted in the DEA environment, such as "production units";
- the seller of the property, the "inputs" (inputs) of its "production unit" are the attributes (intrinsic and extrinsic characteristics) that are relevant to their valuation property, such as location, area, construction standards, etc. The "product" (output) of your "unit" is the price, which can be expressed by Total Asset Value and unit Value (R$ / m^2 of private area);
- for the buyer, conversely, the input is the price, and the outputs are the attributes (location area, etc.);

Thus, they are incorporated, as well, for two optical DEA: the seller and the buyer. For the seller, the goal is to provide some relevant attributes of a property (area, location, etc.). And obtain for them the best possible price. In view of the buyer, the goal is to pay a certain price and in return get the most relevant attributes of a property (better area, better location, etc.).

To collect a sample of the Real Estate Market, we can thus construct two efficient frontiers: the seller's and buyer's:

- the buildings that constitute the efficient frontier of the purchaser are those from relevant attributes, you pay the lowest price;
- the buildings that constitute the efficient frontier of the seller are those where you get a maximum price for a given set of important attributes;
- the buildings included between the two frontiers (data envelopment) are part of the competitive space, and you can design your value in the two efficient frontiers.

Thus, for any given market, with their respective attributes and price values at the borders of the buyer and seller fully justified by the efficient properties in each of the optics can be attached. It becomes possible, therefore, considering a set of market data, to establish a specific property you want to assess what value it should have to be considered "effective" from the point of view of the buyer and what value it should have to be "efficient" from the point of view of the seller, establishing a range similar to the "confidence interval" of statistical inference or "field will" appraiser trading. And it is precisely in this range to be acting with the multicriteria analysis decision AHP

(*Analytic Hierarchy Process*). Then, the mathematical formulation of the problem and the application to a data set, consisting of apartments on offer at the Maracanã neighborhood in Rio de Janeiro, homogeneous in its main features, with the exception of the area and the age will be presented property.

The choice of these data for this study was due to the preliminary presentation of the results obtained with the application of a commercial package in the Brazilian Congress and Expert Reviews. We will seek to indicate how it would be possible to evaluate a property with the use of the DEA double-optics and then compare the results obtained with the linear regression on averages and extremes of attributes property, which we will take on the findings.

Methodology and Mathematical Formulation

The methodological design focuses on quantitative studies. The research methodology classifies the study in terms of purpose, nature, source of data collection, research logic, methodological approach and the intervention instrument used (GIL, 2002; MIGUEL, 2010). Regarding the purpose, the research fits into an applied exploratory study. According to Gil (2002), exploratory studies aim to develop ideas and lead to systematic procedures for empirical observation, as well as enabling the identification of the relationships between the studied phenomena. As for the procedural criteria, it is a multi-criteria analysis with a mathematical modeling instrument. As for nature, it refers to a quantitative study.

The method of envelopment analysis under Dual Optics - EDO-DEA was developed for determining the efficiency of units observed from the DEA Product-oriented and input-oriented classic models, applied simultaneously. As has already been attempted, Products for Product-oriented model are the inputs to the input-oriented model and vice versa ((LYRA (2002)). The formulation of EDO-DEA method for product-oriented (in the case of evaluation, corresponds to the optical vendor) model is the same as the classic DEA model and uses the following formulation of the linear programming problem (CHARNES *et al* (1978)):

Envelope Problem (Seller)

Maximize h, such that:

$X_k \geq \lambda X$

$h Y_k \leq \lambda Y$

$\lambda \geq 0$

$h' = 1/h$

$\sum \lambda = 1$

where:

h = efficiency

X = vector of property attributes (inputs)

X_k = vector of attributes of the property k (inputs)

Y = vector of property prices (product)

Y_k = price of the property k (product)

λ = vector of weights of the linear combination of inputs and products of efficient units

In formulating the EDO-input-oriented DEA model (optical buyer) method transposition of the variables of the classical model is made by exchanging the input X for product Y and vice versa, the resulting formulations (3) and (4) the following linear programming problems - LYRA (2002):

<u>Envelope Problem (Buyer)</u>

Minimize h , such that:

$\lambda X \geq X_k$

$h Y_k \geq \lambda Y$

$\lambda \geq 0$,

$\sum \lambda = 1$

where:

h = efficiency

X = vector of property attributes (product)

X_k = vector of attributes of the property k (product)

Y = vector of property prices (inputs)

Y_k = price of the property k (input)

λ = vector of weights of the linear combination of inputs and products of efficient units

Figure 1 below shows a hypothetical situation graph, where: Y = price and X = area of the property, with the definition of efficient for each of the two optical boundaries.

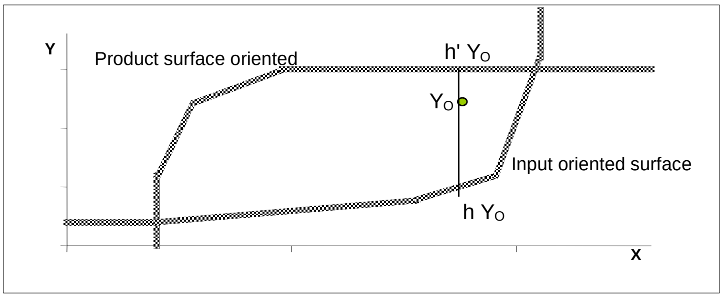

Figure 1 – axes: Y = price e X = area of property

A computer implementation has been developed using the Matlab tool, allowing you to check step by step the search for the solution of the problem described above.

E. *Practical Application - Using EDO-DEA in Evaluation*

N°	Address	Age (years)	VIDR EM	Area (m²)	Price (R$ mil)
1	R. Morais e Silva	10	7697	75	105
2	R. S. Francisco Xavier	30	6599	70	50
3	R. Senador Furtado	18	7110	60	90
4	R. Senador Furtado	18	7110	65	95
5	R. Prof. Manoel de Abreu	25	6781	79	80
6	R. Joaquim Palhares	25	6781	65	65
7	R. Jorge Rudge	25	6781	90	75
8	R. S. Francisco Xavier	30	6599	95	100
9	R. Ibituruna	20	7004	72	105
10	R. Morais e Silva	10	7697	90	130
11	R. Morais e Silva	10	7697	90	115
12	R. Morais e Silva	10	7697	90	120

Table 1 – Sample of real estate market

From the database of Table 1, consists of 12 apartments used in the Maracanã neighborhood and 3 variables (price, area, and apparent age of the property), the following process for applying the DEA method and subsequent identification of values was performed in efficient frontiers of evaluating a hypothetical apartment with 80 m² and 20 years of apparent age:

1st Stage: Preliminarily adjusted variables for the method. The age variable apparent, to have a positive correlation with the price variable was transformed into VIDREM variable, equivalent to (100 * (100-10 * Ln (Age))). The variable area remained unchanged.

2nd Stage: Then, under the efficiencies determined for each optical for 12 flats of the database, using the formulation of the envelope problem in conjunction with widespread simplex (Matlab). From these results, we calculate the maximum and minimum prices corresponding to the projection of each property in efficient frontiers and calculate their IDRF (index of relative distance between boundaries) (see Table 2).

Efficiency (VRS)				Price	Frontier Price		IDRF
Perspective of the seller		Perspective of the buyer		Obs.	Efficients		
Unid.	%	Unid.	%	(R$ mil)	Seller	Buyer	
1	95,45	1	100,00	105.000	110.000,00	105.000,00	4,65%
2	100,00	2	100,00	50.000	50.000,00	50.000,00	0,00%
3	100,00	3	84,00	89.000	90.000,00	75.596,50	17,40%
4	98,57	4	79,58	95.000	96.379,60	75.596,50	24,17%
5	87,41	5	82,03	80.000	91.518,10	65.621,20	32,96%
6	100,00	6	90,95	65.000	65.000,00	59.116,50	9,48%
7	72,27	7	100,00	75.000	103.776,70	75.000,00	32,19%
8	100,00	8	100,00	100.000	100.000,00	100.000,00	0,00%
9	100,00	9	67,04	105.000	105.000,00	70.390,70	39,47%
10	100,00	10	88,46	130.000	130.000,00	115.000,00	12,24%
11	88,46	11	100,00	115.000	130.000,00	115.000,00	12,24%
12	92,31	12	95,83	120.000	130.000,00	115.000,00	12,24%

Table 2 – Results of the implementation in Matlab

3rd Stage: We introduce the apartment evaluating (with an area of 80 m² and 20 years of age, i.e., VIDREM = 7004) in the model (unit No. 13), assuming, as a first attempt, one close to the average price of market data, i.e., R$ 94,000. In the sequel, we apply the methodologies described in the 2nd step. It was observed that the introduction of assessing property in the model was not able to change the efficient frontiers calculated in Step 2, which means that the value of R$ 94,000 is inefficient compared with market data (87.61% of optical efficiency of the seller and 81.08% efficiency in the buyer's perspective). Effective values would be respectively:

- the perspective of the seller: R$ 94.000,00 / 0,8761 = R$ 107.293,69
- the perspective of the buyer: R$ 94.000,00 x 0,8108 = R$ 76.215,20

4th Stage: This step could be necessary if the tentative value of R$ 94,000.00 to alter the structure of the model by modifying any of the borders. In this case, further attempts would be made by increasing or decreasing the initial value of R$ 94,000.00, until there was no changing boundaries calculated for the 12 market data. Reached this point, it would proceed as outlined in the 3rd stage.

As a result of the review, we would have a value range for the property valued between R$ 76,215.20 and R$ 107,293.69. To the apartment in question, the buyer might well justify the low value based on certain market data that contributed to conform the efficient frontier under its optics, as well as the seller could do the same, based on other elements of the same base data.

Comparison of Results with Linear Regression

Was performed, and then with the same database used for the EDO-DEA, multiple linear regression (with the same variables used in the processing of EDO-DEA), in order to compare the estimated results for the values of various properties evaluation:

- Property A: 80 m² and 30 years of age;
- Property B: 80 m² and 20 years of age (even case used in item 3);
- Property C: 90 m² and 10 years of age.

Observe that the property has the attribute lower (older), that equates to the worst of the list of market data, the property is close to B and C of the average conditions, the conditions of the upper roll.

The application of the two proposed methods has led to the following final results:

Property	DEA			Linear Regression		
	perspective buyer	perspective seller	IDRF	lower limit (80%)	upper limit (80%)	IDRF
A	62.502	69.996	11,3%	66.258	83.595	23,1%
B	76.215	107.293	33,9%	85.724	96.374	11,7%
C	115.005	129.993	12,2%	113.173	131.942	15,3%

Table 3 – Comparative Results

Second appoach – a Decision analysus multicriteria - AHP

The Multicriteria Decision Support consists of a set of methods and techniques to assist or support the decisions, when the presence of a multitude of criteria. This process can be broken down into steps (GOMES *et al.*, 2009):

1. Identify the decision makers and their goals;
2. Define alternatives;
3. Define the criteria relevant to the decision problem;
4. Evaluate alternatives against the criteria;
5. Determine the relative importance of the criteria;
6. Achieve the overall evaluation of each alternative;
7. Driving a sensitivity analysis;
8. Propose recommendations;
9. Implement actions.

Steps 1, 2 and 3 are the Structuring Stage, which deals with the formulation of the problem and seeks to identify, characterize, and organize the factors considered important in the decision support process. It is an interactive and dynamic phase because it provides a common language to decision makers, which enables learning and debate. Steps 4, 5, 6 and 7 comprise the Assessment Phase, which aims to apply methods of multicriteria analysis to support the modeling of preferences and their aggregation. The third phase, composed of steps 8 and 9, is Phase Recommendation of the courses of action to be followed.

It should also pay attention to a family of criteria, ie, the set of criteria used in a particular decision situation must meet three conditions ("axioms Roy") to be a coherent family of criteria (ROY and BOUYSSOU, 1993). These axioms, in non-mathematical language, are: Completeness (imposes the need to describe the problem taking into account all relevant aspects); Cohesion (requires the correct analysis of what are the criteria for maximization and minimization which); Not Redundancy (requires the delete criteria to evaluate features already considered by other criteria).

The AHP (*Analytic Hierarchy Process*) is one approach that can be chosen when the decision maker uses its judgment and knowledge to make an assessment of restrictive criteria or not for a given situation. According to Gomes (2009), the AHP is based on peer comparison of criteria, seeking to answer two main questions: What are the most important criteria? What proportion of this importance? To answer these questions, policymakers must assign weights on a scale 1-9 for each criterion, comparing them pairwise (BERZINS, 2009). Gomes (2009) reinforces mentioning that this method can only be used when the parameters have their liabilities are measured on a scale of importance quotient or reason, ie, all parameters should be comparable.

For the case of evaluation of urban properties using EDO-DEA, the application of AHP was performed in the range of values obtained from the viewpoint of the buyer and seller's perspective (Table 3). This technique allowed the decision making of a spot market value for a property assessed, taking into consideration some attributes over others. In the analysis, some assumptions were addressed early in the study, according to previous definitions of the decision maker.

Hypothetical situation

An acquisition of property with 80m^2 and apparent age of 20 years. What punctual market value for this property which is with the variation of R$ 76,125 to R$ 107,293 at the borders of the buyer and seller respectively? The central idea is to find value for x_1 and x_2 such that:

$$Value\_property = 76125x_1 + 107293x_2 \quad 0 \leq x_1, x_2 \leq 1 \quad e \quad x_1 + x_2 \leq 1$$

For the analysis of Table 3, we can draw the following changes in relation to the attributes age and built to the property reference depreciation area.

Variation Percentage of attributes		
	Age	**Area**
perspective Seller	34,8 %	21,17 %
perspective Buyer	18 %	50,89 %

Table 4 – Representation of the percentage variation between the attributes of the property according to frontier EDO-DEA

Taking as criteria for prioritization age in relation to the area, we can see that, for the buyer, the difference from the value of the property is much smaller than the area considered. However, if the buyer prioritizes the area, the "leap" in value relative to the acquisition of the property is more expressive, while for the seller, this difference is reasonable. Thus, by assigning weights to the criteria age = 6 (0.75 normalized) and area = 2 (0.25 normalized) the hypothetical situation in question in the hierarchy of criteria / objectives of the AHP tree, leads to the optical frontier the buyer (lower property value).

So, a suggestion to get off the appraised value of the property, with the characteristics of area = 80 m^2 and 20 years of age (VIDREM = 7004) is as follows:

$$Value\_property = 76125x_1 + 107293x_2 \quad 0 \leq x_1, x_2 \leq 1 \quad e \quad x_1 + x_2 \leq 1$$

$x_1 = 1$ and $x_2 = 0$ (result on the frontier of the buyer)

$$Value\_property = 76125$$

Applying the correction in property value according to decision criteria and considering the IDRF, we have:

$$Mercadologic\ Value\_property = 76125 + \frac{IDRF}{2}.(0{,}75.76125 + 0{,}25.107293)$$

$$= 90348{,}93$$

Therefore, the fair value of property assessment would be R$ 90,348.93.

Conclusion

It can be seen by examination of Table 3 that the implementation of EDO-DEA allows to achieve more closed intervals for projections at the ends of the sample frontier, while the application of multiple regression reaches over closed intervals for evaluations of real estate with the closest attributes average conditions of the sample. The application of this methodology in the evaluation of properties was possible only after the use of computer system (Matlab) with the use of optimization methods and decision, which was adapted for the case study. The results obtained using the EDO-DEA and Multicriteria Decision Analysis shows that new methodologies allow the Engineer Reviewer obtaining reliable results, and that can be improved for future application, bringing promising benefits for the area of evaluation of urban real estate. Further investigations using only multi-criteria decision analysis (AMD) are suggested to compare the results with traditional statistical inference and the proposed EDO-DEA with decisor.

References

ASSOCIAÇÃO BRASILEIRA DE NORMAS TÉCNICAS. **ABNT NBR 14653:** Avaliação de bens – Parte 1: Procedimentos gerais. Rio de Janeiro, 2019.

ASSOCIAÇÃO BRASILEIRA DE NORMAS TÉCNICAS. **ABNT NBR 14653:** Avaliação de bens – Parte 2: Imóveis Urbano. Rio de Janeiro, 2011.

BERZINS, L.J. **Avaliação de Desempenho pelo AHP, através do superdecisions:** caso INMETRO. 2009. Dissertação (Dissertação de Mestrado Profissionalizante em Administração), Faculdades Ibmec, Rio de Janeiro, 2009.

CHARNES, A.; COOPER, W.W.; RHODES, E. Measuring the efficiency of decision making units. **European Journal of Operational Research**, Vol.2, Issue 6, November

1978, p.429-444. Disponível em: https://doi.org/10.1016/0377-2217(78)90138-8. Acesso em 06 mai 2021.

DANTAS, R. A. **Engenharia de Avaliações:** uma Introdução à Metodologia Científica.São Paulo: PINI, 2001.

DANTAS, R. A. **Ingeniería de Tasaciones:** una Introducción a la Metodología Científica. Venezuela: Novo Horizonte, 2002.

FÄRE, R.; GROSSKOPF, G. Estimation of returns to scale using data envelopment analysis a comment. **European Journal of Opertion Research**, Vol.79, Issue 2, 8 December 1994, p. 379-382. Disponível em: https://doi.org/10.1016/0377-2217(94)90366-2. Acesso em: 12 jun 2021.

FÄRE, R.; GROSSKOPF, G; LOVELL, C.A.K. **Production Frontiers**. Cambridge: Cambridge University Press, 1996.

FÄRE, R., LOVELL, C.A.K.; ZIESCHANG, K. Measuring the technical efficiency of multiple output technologies. In: Eichhorn, W.; Henn, R.; Neumann, K.; Shephard, R. W. **Quantitative studies on production and prices**. Springer, 1983. p. 159-171.

FÄRE, R.; GROSSKOPF, S.; LOVELL, C.A.K. **The Measurement of Efficiency of Production**. Boston: Kluwer-Nijhoff Publishing, 1985.

FARRELL, M.J. The Measurement of Productive Efficiency. **Journal of the Royal Statistical Society**. Series A, Vol. 120, n.3, 1957, p. 253-281. Disponível em: https://doi.org/10.2307/2343100. Acesso em: 04 mai 2021.

GIL, A. C. **Como elaborar projetos de pesquisa**. São Paulo: Atlas, 2002.

GOMES, L.F.A.M.; GONZALEZ-ARAYA; M.C., CARIGNANO, C. **Tomada de decisões em cenários complexos**. Rio de Janeiro: Pioneira Thompson Learning, 2009.

LINS M.E.; MEZA, L.A. **Análise Envotória de Dados e perspectivas de integração no ambiente de Apoio à Decisão**. Rio de Janeiro: Coppe/UFRJ, 2000, p. 7-53.

LINS, M.E; LYRA, L.F.N.; LEGEY, L.F,L.DSV DEA na avaliação imobiliária em ambiente SIG, **XI Claio**, 2002.

LYRA, L.F.N. **Envoltória sob Dupla Ótica aplicada na Avaliação Imobiliária em ambiente do Sistema de Informação Geográfica**. Tese. VII, 85 p., COPPE, UFRJ, Engenharia de Produção, 2002.

MIGUEL, P. A. C. (organizador). **Metodologia de pesquisa em engenharia de produção e gestão de operações**. Rio de Janeiro: Elsevier, 2010.

ROY, B.; BOUYSSOU, D. **Aide multicritère à la décision: méthods et cas.** Paris: Ed. Economica, 1993.

SHEPHARD, R.W. **Cost and production function**. New Jersey: Princenton University Press, 1953.