

## Frege on Singular Senses<sup>1</sup>

### *Frege sobre os sentidos singulares*

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**Abstract:** In this article the author discusses what seems to be a puzzle for Frege's notion of singular senses (i.e., the senses of singular terms), in particular senses of definite descriptions. These senses are supposed to be complete (or saturated), but they are composed of the incomplete (unsaturated) senses of conceptual terms (i.e., conceptual senses). The author asks how the definite article (or what it expresses) transforms an unsaturated sense into a saturated one and reviews some attempted explanations in the literature. He argues that none of them is compatible with Frege's views in semantics. Next, he discusses an alternative that Frege himself endorses and argues that it is also incompatible with his semantics. The author concludes that Frege has no coherent view on the senses of definite descriptions. If we assume that every name expresses a descriptive sense, then we must conclude that Frege has no coherent explanation for singular senses in general.

**Keywords:** Frege, definite article, proper names, reference.

**Resumo:** Neste artigo, o autor discute o que parece ser um enigma para a noção de Frege de sentidos singulares (ou seja, os sentidos dos termos singulares), em particular os sentidos das descrições definidas. Estes sentidos devem ser completos (ou saturados), mas são compostos pelos sentidos incompletos (insaturados) dos termos conceituais (i.e., dos sentidos conceituais). O autor pergunta como o artigo definido (ou o que ele expressa) transforma um sentido insaturado em saturado e revisa algumas tentativas de explicação na literatura. Ele argumenta que nenhuma delas é compatível com a posição de Frege na semântica. Em seguida, ele discute uma alternativa que o próprio Frege endossa e argumenta que ela também é incompatível com sua semântica. O autor conclui que Frege não tem uma visão coerente sobre os sentidos das descrições definidas. Se assumirmos que cada nome expressa um sentido descritivo, então devemos concluir que Frege não tem uma explicação coerente para os sentidos singulares em geral.

**Palavras-chave:** Frege, artigo definido, nomes próprios, referência.

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<sup>1</sup> Artigo publicado originalmente na *Grazer Philosophische Studien*, v. 98, issue 2, 2021, pp. 316-339. © Koninklijke Brill NV, Leiden, 2020, e aqui republicado com permissão dos editores. DOI da publicação original: 10.1163/18756735-00000118

The purpose of this paper is both historical and critical: I intend to discuss Frege's view on the senses ("Sinn") of singular terms (which I will call singular senses) as opposed to senses of conceptual expressions (which I will call conceptual senses), i.e., how they are formed and what their structure is. On the one hand, I will stay as close as possible to Frege's writings and try to extract his exact view from them. On the other hand, I will argue that a surprising incoherence appears along the way. I start by reviewing some well-known elements of Frege's doctrine of sense and reference. As I intend to show, when these elements are brought together in a non-standard way, a puzzle seems to emerge regarding singular senses: where does their saturation come from? There are some proposals in the literature on Frege or in theories inspired by Frege and I shall review the most significant, but each of them seems either to be directly incompatible with other important aspects of Frege's doctrine or to betray the spirit of the Fregean notion of sense. Frege says remarkably little about the senses of singular terms; one can find only a few scattered remarks about this topic in his writings. But some things that he does say suggest an approach to singular senses that is blatantly incompatible with the rest of his semantics. So, my conclusion will be that, for all the elegance and epistemic advantages of the Fregean notion of sense, he has no coherent view on singular senses. This might be alarming, for these are supposed to be the most basic senses from an epistemic point of view. And, given the complementarity of singular senses and conceptual senses, the incoherence might infect the latter notion as well (and, a fortiori, the notion of sense as a whole).

## 1 PROPER NAMES

According to the standard interpretation of Frege's doctrine of sense and reference, any singular term essentially expresses a sense ("otherwise it would be an empty sequence of sounds and it would be wrong to call it a name" (NS, p. 135/PW, p. 124)), and only accidentally has a reference (i.e., might have it or not).<sup>2</sup> The reference of a singular term (if it has one) is the object uniquely "determined" by the sense. Frege characterizes senses as containing the "modes of presentation" of the reference (1892, p. 26). He employs the famous metaphor of saturation/unsaturation to elucidate a categorical distinction between singular senses and conceptual senses. The senses of singular terms like 'Lisbon', 'the capital of Portugal' and 'this city' and of complete sentences are, according to him,

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<sup>2</sup> It is less clear whether a predicate or conceptual word can express a sense without having a reference.

complete or saturated, while the senses of conceptual terms like ‘city’ or ‘capital’ or of predicates like ‘is a city’ or ‘is a capital’, as well as of functional expressions like ‘the capital of’ are incomplete or unsaturated.

Frege also defends on many occasions (e.g., in a famous letter to Husserl (WB, p. 96/PMC, p. 63)) the thesis (which is, strictly speaking, independent from the previous one) that, as regards saturation/unsaturation, there is a kind of isomorphism between the levels of language, sense and reference. According to this thesis, complete or saturated expressions have saturated senses, and thereby refer (if they have a reference at all) to complete or saturated entities (ordinary objects or truth-values). Unsaturated expressions have unsaturated senses and refer to unsaturated entities (first or higher-order functions, concepts, etc.). Frege considers ordinary proper names, complex demonstratives and definite descriptions as belonging to the broad category of proper names (“Eigennamen”). In many passages he indicates that he takes this category as including any expression (simple or complex) that is meant to stand for a single object (e.g., 1892, p. 27; NS, p. 107/PW, p. 97, n. 2; NS, p. 262/PW, p. 243; NS, p. 193/PW, p. 178; WB, p. 154/PMC, p. 96). He never explicitly states that one of these categories of names is more basic or paradigmatic in the sense that all other categories could be seen as disguised forms of it. But the few illustrations that he provides of senses of proper names are descriptive senses. One occurs in a famous footnote (1892, p. 27) in which he comments that people might associate different senses with a name like ‘Aristotle’, and gives as examples “the pupil of Plato and teacher of Alexander the Great” and “the teacher of Alexander the Great who was born in Stagira” (1892, p. 28). Another occurs in a similar remark about the understanding that two persons might have of a name like ‘Dr. Gustav Lauben’ as “the doctor who is the only doctor living in the house known to both of them” (1918–19, p. 65). Based on these passages, many contemporary readers or critics of Frege (e.g., Kripke (1980, p. 27), Kaplan (1989, p. 516), Soames (2005, p. 9), among many others) interpret him as holding what is called a descriptivist theory of singular terms, i.e., the view that ordinary proper names and demonstratives have a descriptive sense, which means that they express the same sense as some definite description. However, another group of readers think that there is not enough evidence to see him as holding this view, especially in the case of demonstratives (e.g., Dummett (1973, p. 110, p. 135), Carl (1994, p. 184), Evans (1982, p. 18) and McDowell (1977)). Evans in particular thinks that some room is left in Frege’s theory for non-descriptive singular senses. A prototypical case of a non-descriptive sense would be that of the first-person pronoun ‘I’ which, according to Frege (1918–19, p. 66), whenever

thought by a person contains a “special and primitive” form of presentation of the same person, and this form of presentation is not captured by any description of the form ‘the person so and so’.

There are, hence, two alternative interpretations. According to the first, Frege reduces all singular terms to implicit forms of definite descriptions. According to the second, there might be singular terms that are not so reducible and, hence, even if we get a clear understanding of the nature of definite descriptions, we might not necessarily know what the senses of ordinary proper names and complex demonstratives are. I think that the first interpretation has stronger support in Frege’s few examples of singular senses, but I shall not dwell on this. What follows in this paper does not depend on accepting this interpretation. If the first interpretation is correct, then the problem that I will discuss generalizes to the totality of singular senses. But if it the second interpretation is correct, then the problem is primarily one for definite descriptions, and that is enough of a problem since definite descriptions are one of the fundamental referring expressions for Frege. And this means that an entire tradition of interpreting Frege along the lines suggested by Strawson (1950) has to be seen as highly problematic.<sup>3</sup>

## **2 A PUZZLE: WHAT SATURATES THE SENSE OF A PROPER NAME?**

Whatever the correct interpretation of ordinary proper names is, the fact is that singular senses are supposed to be saturated, and in this respect they all belong to the same category. In some passages Frege insists that the thought corresponding to a singular judgment (i.e., a thought that a particular object has a certain property) is composed of parts corresponding to the syntactic parts of the sentence expressing it. One part (the one corresponding to the predicate) is unsaturated, and the other (corresponding to the name) is saturated (NS, p. 203/PW, p. 187; NS, p. 217/PW, p. 201). Let us look inside the sense of a definite description and consider how it is composed. The prototypical form of a definite description is ‘the F’, where ‘F’ is the name of a simple or compound concept and,

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<sup>3</sup> Another (related) issue that I shall bypass in this paper is whether proper names can express senses in the absence of a reference. Some authors, most notably McDowell (1977, pp. 172–73) and Evans (1982, p. 22), interpret the Fregean semantics as having the notion of reference as primary and the notion of senses as derivative. Hence, they claim, we should not take Frege as allowing proper names with a sense independently of having a reference. Fictional names, for instance, which do not have a reference, have only a pseudo-sense. Sentences of which a part lacks a reference express only “mock-thoughts”. I think this is incorrect and that the evidence strongly supports the interpretation of senses existing in the absence of reference. I agree with Bell (1990) that Evan’s interpretation is ultimately untenable and renders Frege’s views on sense and reference inconsistent and largely ad hoc. But I shall leave this issue aside here. I will be interested only in the structure of singular senses, whether or not they are necessarily connected with a reference.

hence, has a sense as well. One important question to be raised is whether there is any difference between the sense of 'F' and the sense of 'the F', i.e., whether the addition of the definite article essentially changes the sense expressed. Of course, there must be a difference, since the former has a concept as reference, and the latter has an object. Although different senses might have the same reference, there cannot be one sense with different references. Moreover, the former has, according to Frege, an unsaturated sense, while the latter has a saturated one. An unsaturated sense can never be the same as a saturated one. If this is so, the difference between the sense of 'the F' and of 'F' can only be due to the semantic contribution of the definite article. But what exactly is the semantic contribution of the definite article that transforms the unsaturated sense of 'F' into the saturated sense of 'the F'? Where does saturation come from? This, I think, poses a puzzle for the interpretation of Frege's notion of singular senses of definite descriptions. In trying to solve the puzzle, we have to keep in mind a principle that is at the bottom of Frege's semantics, namely:

*(P) A complex sense is composed only of senses.*

Frege assumes or explicitly endorses (P) on several occasions (e.g., WB pp. 127–8/PMC pp. 79–80; WB p. 86/PMC p. 55; WB p. 231/PMC p. 149; WB p. 245/PMC p. 163; NS pp. 208–9/PW pp. 191–2; 1892, p. 33; 1892a, p. 205). (P) implies that no complex sense can result from the combination of simple senses with things that are not senses. Hence, it rules out combinations of senses and extensions (objects, etc.) as composing “hybrid” senses. An important corollary of (P) is the following:

*(P\*) Only a sense can saturate another sense.*

This precludes, e.g., the possibility that an object like Lisbon can saturate the sense of 'is a capital' to produce a complete sense (the thought that Lisbon is a capital). Only a sense, e.g., the sense of 'Lisbon' can do that. But it also precludes the possibility that anything like a psychological attitude (intention, belief, speech act or something of the sort) can saturate the unsaturated sense. This will be relevant in what follows.

### **3 SOME NON-SOLUTIONS**

Before discussing some possible solutions (or non-solutions) for the puzzle, it might be useful to review the conditions that Frege places on the acceptability of definite descriptions (or, more broadly speaking, on the correct use of the definite article) in scientific contexts. There are, according to him, two crucial conditions for expressions of the form 'the F' to have a reference: first, there must be at least

one object instantiating F (existence) and, second, there must be at most one object instantiating F (uniqueness). Sometimes Frege mentions only existence as a requirement (e.g., in GLA, §97) and sometimes only uniqueness (e.g., in 1892, p. 5; GGA II, §100; WB, p. 195/PMC, p. 127; NS, p. 178/PW, p. 163), but most of the time he stresses both existence and uniqueness (e.g., in GLA, §74, n.1; 1892a, p. 204; WB, p. 89/PMC, p.58; WB, 154/PMC, p. 96; NS, 193/PW, 178). It seems that he gives equal weight to existence and uniqueness as conditions for the legitimate use of the definite article and nowhere, as far as I know, recognizes any asymmetry between the two conditions. So, apparently, what goes for one condition goes for the other as well.<sup>4</sup>

Let us now go back to the puzzle of how senses of conceptual words in definite descriptions get saturated. There is a more or less natural answer to the puzzle, in the spirit of (though not identical with) Russell's classical analysis of definite descriptions: the definite article attached to a conceptual term expresses the two conditions above (existence and uniqueness), i.e., it contributes to the sense of the definite description with a sense of a second-order function corresponding to the conjunction of the existence and uniqueness clause. In other words, the definite article in 'the F' has as reference the conjunction of the second-order function corresponding to 'there is one x such that Fx' with the second-order function corresponding to 'there is at most one x such that Fx', and its sense is the conjunction of the senses for these second-order functions.

This is a convenient way of accommodating the sense of the definite article. But it is incompatible with Frege's view. First, the sense of a predication with a definite description as subject turns out to be not that of a singular thought (as Frege thinks it should be), but of an existential thought. Frege regards it as essential to definite descriptions that they are proper names, i.e., they are at least meant to be referential devices (although they may fail in referring). Second, and more importantly, Frege explicitly rejects the thesis that existence is part of the thought expressed by a sentence with a definite description. E.g., in the following famous passage of "Über Sinn und Bedeutung" (1892), in which he discusses the sentence 'The one who discovered the elliptic form of the planetary orbits died in misery' (1892, p. 40):

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<sup>4</sup> One might be tempted to read Frege as holding a view similar to Hilbert and Bernays' (1934, §8) according to which the definite article can be used only if a proof of existence and uniqueness is available. However, Frege does not hold this view either in his theory of language or in his formal system. The operator that replaces the definite article in the formal system of GGA I can be placed in front of any argument (name of object) without any proof of existence or uniqueness.

Now languages have the fault of containing expressions which fail to designate an object (although their grammatical form seems to qualify them for that purpose) because the truth of some sentence is a prerequisite. Thus it depends on the truth of the sentence 'There was someone who discovered the elliptic form of the planetary orbits' whether the subordinate clause 'The one who discovered the elliptic form of the planetary orbits' really designates an object, or only seems to do so while in fact is bedeutungslos. And thus it may appear as if our subordinate clause contained as a part of its sense the thought that there was somebody who discovered the elliptic form of the planetary orbits. If this were right, the negation would run:

'Either the one who discovered the elliptic form of the planetary orbits did not die in misery or there was nobody who discovered the elliptic form of the planetary orbits'.

The last sentence is clearly meant as a *reductio*, since the negation of 'The one who discovered the elliptic form of the planetary orbits died in misery' is not the one presented.<sup>5</sup> That is to say, existence is not, in his view, part of the thought expressed by a sentence containing a definite description. As I will discuss soon, existence is only a presupposition that has to be true if the thought is to be true or false. In this passage, Frege mentions only existence, and is silent about uniqueness. According to modern linguistics, it is not incoherent to treat definite descriptions as presupposing existence and expressing uniqueness (hence, considering both conditions as having different weights). But this hardly seems to be Frege's view since, as we saw, he usually considers the two conditions on a par, and nowhere mentions any asymmetry between them. So, I take it that this remark (and others to the same effect that I will quote later) about existence not being part of the thought expressed are meant to apply to uniqueness as well, i.e., uniqueness is not part of the thought expressed either.

Chateaubriand (2001, 2005, 2007) suggests that the sense of a definite description could be a property that contains uniqueness as part of its content. The sense of a singular term like 'the F' would be, according to this proposal, a property of the form

$$Fx \wedge (\forall y)(Fy \rightarrow y=x).^6$$

This proposal is, like the previous one, incompatible with the view that uniqueness is not part of the content (as I said, this is most likely Frege's view assuming that he considers uniqueness on a par with existence).

Another suggestion could be formulated adopting a Carnapian representation of intensions as functions from possible worlds into the set of

<sup>5</sup> As Atlas (1975, p. 31) points out, in this argument Frege seems to assume that the only way in which the thought of existence could be part of the overall thought would be as one of its conjuncts.

<sup>6</sup> Although this formula is a conjunction, it should be read as a property of one variable ('x'). We could also represent it using lambda-notation as  $(\lambda x)[Fx \wedge (\forall y)(Fy \rightarrow y=x)]$

(appropriate) extensions. The sense of a definite description can be represented as a partial function from the set of possible worlds into the set of objects. (If a possible world receives a value by the function, of course the value has to be an object existing in that world. There might be worlds where there is no object satisfying the description, and hence the function assigns no value to that world.) The sense of a predicate might be seen as a function associating a set of objects to each possible world. The difference between both senses would be a functional one. The sense of the definite article could then be seen as a second-order function from the set of functions of the second kind (i.e., senses of predicates) into the set of functions of the first kind (i.e., senses of definite descriptions). Both Chateaubriand's and the Carnapian representation of senses deviate from Frege's proposal in two fundamental aspects. First, the senses under consideration would be functions and, as such, unsaturated entities according to Frege's ontology, while the senses of definite descriptions are saturated. Second, and more importantly, in Frege's view senses cannot be concepts. Concepts are functions, and functions are too coarse grained to explain the cognitive dimension of predicates. E.g., two concepts like *successor of 1* and *prime even number* univocally instantiated by the same object would count as the same sense according to both proposals; but this is incompatible with the spirit of the Fregean notion of sense, which is supposed to play an epistemic role.

Dummett (1973, p. 94 and p. 530) regards the definite article in definite descriptions as a second-order operator on a par with, e.g., the abstraction operator (as in 'the class of  $x$ s such that  $Fx$ '), and the numerical operator (as in 'the number of  $x$ s such that  $Fx$ '). According to this suggestion, the definite article has as reference a second-order function that associates with any concept  $F$  the one and only object that is  $F$ . (Presumably it has to be a partial function, for there are concepts that do not have one unique instance, and in these cases the function is undefined; or else, the function can be completed by stipulation, i.e., by associating any arbitrarily chosen object with those concepts that fail to have one and only one instance.) One could further explore Dummett's parallel of the definite article with the other operators. In the case of 'the class of  $x$ s such that  $Fx$ ' we know, as a result of Russell's paradox, that not all concepts generate a corresponding class, and hence this expression only has a reference if the concept generates a class. However, the claim that  $F$  generates a corresponding class does not seem to be part of the content of a sentence containing 'the class of  $x$ s such that  $Fx$ ': this claim is simply a presupposition that one makes in order to use the term referentially. Something similar happens in the case of the numerical operator: for



a concept  $F$  to have a corresponding number, according to Frege, it must have sharp delimitation, i.e., it must yield a truth-value for any object as argument. But we would hardly regard the thought expressed by a numerical equation like ‘the number of  $x$ s such that  $Fx$  is identical with 5’ as including the clause that  $F$  is sharply delimited. As it happens in the case of the abstraction operator, there are conditions for the numerical operator to generate a referential singular term when attached to a predicate, but these conditions do not necessarily show up in the content expressed by the singular term, or by a more complex expression (e.g., a sentence) containing the singular term as part. Whenever we say something using ‘the class of  $x$ s such that  $Fx$ ’ (or ‘the number of  $x$ s such that  $Fx$ ’) we presuppose that  $F$  generates such a class (or is sharply delimited), but this is not part of our thought. We could perhaps say the same about definite descriptions: there are two conditions for definite descriptions to be referential singular terms, namely, existence and uniqueness, but, as in the case of the abstraction and numerical operators, these conditions do not have to show up in the content expressed by the description.

There is a serious drawback with Dummett’s interpretation, however. If the definite article corresponds to a function that simply assigns an object to the concept  $F$ , but it is not part of its sense that the object assigned is the one and only  $F$ , then we have a barren explanation of the saturation of senses in definite description. This attribution of an object to the property, without any rule or specification, seems entirely arbitrary and *ad hoc*. If there must be an intuitive and natural connection between the sense of ‘ $F$ ’ and the sense of ‘the  $F$ ’, then the sense of the definite article must somehow contain an instruction to pick the one and only  $F$ , and not simply give us as if “by magic” the one and only  $F$ . Otherwise there would be no principled reason for regarding as the correct reference of the definite article the function that gives the one and only  $F$  to  $F$  instead of, say, the function that gives any other arbitrary non- $F$  to it. In other words: the solutions suggested by Dummett can only yield an interesting explanation of the saturation of the incomplete sense expressed by ‘ $F$ ’ if the definite article has as reference the second-order function that he suggests, and additionally has a sense that somehow embodies existence and uniqueness as the rule behind this function. But this is precisely something that Frege denies in the famous passage of “*Über Sinn und Bedeutung*” (1892, p. 40) quoted above.

Dummett also draws a contrast between the sense of proper names and the sense of predicates in terms of the discriminatory knowledge that is required for grasping each of them (1973, p. 229). The contrast, however, does not help us with

the puzzle. For we still need an explanation of what turns a criterion for deciding whether any object has a property (or a set of properties) into a criterion for discriminating a particular object as the referent of a name. For instance, I may be able to decide whether any person (including Obama) has the property corresponding to ‘president of the US reelected in 2012’. But something else is necessary for me to say that Obama is the referent of ‘the president of the US reelected in 2012’. Simply having a property (or a set of properties) is not in itself a reason to consider one particular object as the referent of a name unless there is something special about that object (like being the only one with that property). Uniqueness is not given in the sense of a property (except when we have uniqueness as part of the form of the property, but this only happens for special properties, and cannot be assumed as a general solution to the puzzle). Hence, to consider Obama as the referent of ‘the president of the US reelected in 2012’ we need to add something to the content of ‘president of the US reelected in 2012’, and that something, as it seems, has to be the contribution of the definite article.

None of the alternatives reviewed so far gives a solution to the puzzle: we still have no account of what bridges the gap between unsaturated conceptual senses and saturated senses of definite descriptions. These alternatives are adopted by Fregean scholars or Frege-inspired philosophers, but Frege himself nowhere endorses any of them, and they are in fact incompatible with his broader views on senses and definite descriptions. In the next section I will discuss an alternative that Frege does suggest and endorses (and, as it seems, never gives up). But, as I shall argue, it is also incompatible with Frege’s semantics.

#### 4 THE PRESUPPOSITION-INDICATOR INTERPRETATION

Although Frege considers that existence is not part of the official content of a thought expressed by a sentence containing a definite description, he says on a number of occasions that it is a *presupposition* (“*Voraussetzung*”) of the use of the description in contexts in which truth and falsity matter.<sup>7</sup> Contemporary linguistic theory standardly distinguishes two notions of presupposition, namely, semantic and pragmatic presupposition (e.g., Keenan (1971), Karttunen (1973) and Levingson (1983)). The semantic notion is sometimes explained in terms of truth: a sentence (or utterance) A semantically presupposes a sentence (or utterance) B

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<sup>7</sup> As I said, he most likely thinks the same about uniqueness, although there is no decisive textual evidence for this.

iff (i) if A is true, then B is true and (ii) if A is false, then B is true.<sup>8</sup> The pragmatic notion is sometimes explained (e.g., by Stalnaker (1970)) in terms of the beliefs that the speaker of an utterance takes for granted and assumes that the audience does the same. In the semantic notion, it is a sentence (or utterance or proposition) that has presuppositions, i.e., presupposition is a relation between sentences (or utterances or propositions); in the pragmatic conception, it is a speaker who has presuppositions, i.e., presupposition is a relation between the speaker and sentences (or utterances or propositions). In the several passages in which he talks about presuppositions Frege does not seem to distinguish between these different notions, and sometimes has the semantic and sometimes the pragmatic notion in mind.<sup>9</sup> E.g., in the following passages, he seems to have semantic presupposition in mind.

The first one was already quoted (Frege 1892, p. 40):

Now languages have the fault of containing expressions which fail to designate an object (although their grammatical form seems to qualify them for that purpose) because the truth of some sentence is a prerequisite. Thus it depends on the truth of the sentence 'There was someone who discovered the elliptic form of the planetary orbits' whether the subordinate clause 'The one who discovered the elliptic form of the planetary orbits' really designates an object, or only seems to do so while in fact is *bedeutungslos*. And thus it may appear as if our subordinate clause contained as a part of its sense the thought that there was somebody who discovered the elliptic form of the planetary orbits. If this were right, the negation would run:

'Either the one who discovered the elliptic form of the planetary orbits did not die in misery or there was nobody who discovered the elliptic form of the planetary orbits'.

The second one is this (Frege 1892, 42 n. J):

The sense of the sentence 'After Schleswig-Holstein was separated from Denmark, Prussia and Austria quarreled' can be rendered in the form 'After the separation of Schleswig-Holstein from Denmark, Prussia and Austria quarreled'. In this version, it is surely sufficiently clear that the sense is not to be taken as having as part the thought that Schleswig-Holstein was once separated from Denmark, but that this is the necessary presupposition in order for the expression 'after the separation of Schleswig-Holstein from Denmark' to have a *Bedeutung* at all.

But in the following passages he seems to have pragmatic presupposition in mind since he talks about things that the speaker does while uttering names or assertions.

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<sup>8</sup> This requires a distinction between logical consequence and entailment, which is a weaker relation. A sentence entails its presupposition but does not imply it logically.

<sup>9</sup> Atlas (1975) points out this ambiguity in Frege's notion of presupposition. He actually detects a third notion of presupposition that he calls assertoric, which can be seen as a relation between the illocutionary act of assertion and propositions. In many passages in which pragmatic presupposition is intended, Frege talks about what is involved in making assertions. This indicates, I think, that he does not distinguish pragmatic from assertoric presuppositions either.

For instance (Frege 1892, pp. 31–2):

[B]ut how do you know that the name ‘The Moon’ has any *Bedeutung*? How do you know that anything whatsoever has a *Bedeutung*? I reply that when we say ‘The Moon’, we do not intend to speak of our idea of the Moon, nor are we satisfied with the sense alone, but we presuppose a *Bedeutung* [...] Now we can of course be mistaken in the presupposition, and such mistakes have indeed occurred [...] in order to justify speaking of the *Bedeutung* of a sign, it is enough, at first, to point out our intention in speaking or thinking.

Or (Frege 1892, p. 40):

If anything is asserted there is always an obvious presupposition that the simple or compound proper names used have a *Bedeutung*. If therefore one asserts ‘Kepler died in misery’, there is a presupposition that the name ‘Kepler’ designates something; but it does not follow that the sense of the sentence ‘Kepler died in misery’ contains the thought that the name ‘Kepler’ designates something.

In the first passage the notion of presupposition is explained in terms of the speaker’s intention, and in the second in terms of what is required for the speech act of assertion. Frege is not referring here to a relation that exists between thoughts but rather to something that speakers do, i.e., pragmatic presupposition. Since Frege makes no distinction between semantic and pragmatic presupposition and seems to oscillate between both notions, I take him to assume that both go together, i.e., every semantic presupposition of a thought P (i.e., what is required for P to have a truth-value) is at the same time a pragmatic presupposition (i.e., something that an ideal speaker believes or takes for granted) in asserting or simply considering P in a context in which truth-value matters.<sup>10</sup>

Is there any relation between the definite article in a definite description and the presupposition (semantic or pragmatic)? Frege seems to suggest that the presence of the definite article plays the role of an indicator of the presupposition without thereby having it as meaning. In other words, the definite article marks a presupposition or an attitude on the speaker’s part but has no meaning of its own. There are a number of passages in which the vocabulary employed by Frege suggests that he has something along these lines in mind. Let us look at some of them.

The first passage I want to consider is from “*Über Schoenflies: Die logischen Paradoxien der Mengenlehre*” (1906), in which Frege explicitly connects the presupposition with the use of the definite article (NS 194/PW 179):

[G]iven a concept, you may not presuppose without further question that an object falls under it. But someone does that [“das tut man aber”] when, by means of a

<sup>10</sup> The converse might not be true, i.e., there might be pragmatic presuppositions (beliefs) that are not semantic presuppositions. This is in accordance with the contemporary view on presuppositions (e.g., Stalnaker (1970)).

definite article or demonstrative, he forms out of a concept-word a proper name destined for use in science.

That is to say, the definite article in proper names (definite descriptions) for use in science indicates something that the speaker does, i.e., the presupposition. The definite article indicates that an object falls under the concept but cannot express it as part of its sense since, as we saw from the passage of “*Über Sinn und Bedeutung*” (1892, p. 40), existence is not part of the thought expressed by a sentence containing it.

The next two passages are important because of the vocabulary that Frege employs when describing the presence of the definite article. The first one is from GLA §66 in which he justifies taking the expression ‘the direction of a’ as a proper name. The definite article plays a crucial role here (my emphasis.): “In the proposition “the direction of a is identical with the direction of b” the direction of a plays the part of an object. [Footnote]: This is *shown* by the definite article [Der bestimmte Artikel deutet dies an]”.

Frege uses the verb ‘*andeuten*’, which Austin translates as ‘show’, but this might be misleading, since its meaning in German is weaker than that (at least in one connotation of ‘show’). ‘*Andeuten*’ is closer to ‘give a hint’ or ‘give a signal’, which suggests that the definite article does not mean, but only indicates something. The same suggestion can be read, as I see it, in a footnote that occurs only in the unpublished version of “*Über Begriff und Gegenstand*” (which the editors of Frege’s *Nachlass* placed together with the published version), where he remarks (NS p. 110/PW p. 100; my emphasis.): “The definite article does not add a new characteristic mark. What it does do is to indicate [liegt in ihm angedeutet]: (1) That there is such a result. (2) That there is only one such.”

Here again the same verb (‘*andeuten*’), i.e., the same careful characterization that occurs in the passage from the footnote of GLA §66, indicates something weaker (from the semantic point of view) than ‘*bedeuten*’. Just some few lines below (in the same footnote) Frege adds a remark that seems to pull in the opposite direction, i.e., of the definite article contributing to the sense of the definite description (NS p. 112/PW p. 102):

It is, accordingly, clear that the definite article makes an essential contribution [wesentlich ... mitbestimmt] to the sense of our sentence and that the position here is consequently quite different from what it is in the case of ‘The capital of the German Empire is the city of Berlin’, where if you leave out ‘the city of’ you are only suppressing a qualification which is incidental to the sense.

The German verb *'mitbestimmt'* (which might be understood in some different ways, e.g., 'helps to determine', 'has a voice', etc.) was translated by Geach as 'makes a contribution', thereby suggesting that the definite article expresses a sense that combines with the sense of the conceptual term to build the singular sense of the definite description. But the passage doesn't have to be taken in the way suggested by Geach's translation, especially if we consider the context in which it occurs. Frege is arguing against Kerry's contention that the expressions 'the result of additively combining 3 and 1' and *'result of additively combining 3 and 1'* belong to the same category (which means, according to Kerry, that the number 4 and the concept result of additively combining 3 and 1 are the same). Frege criticizes Kerry for not taking into account the presence of the definite article as an indicator of a categorical difference between both expressions. But there is no indication in the text (except for the translation that Geach chooses for *'mitbestimmt'*) that the categorical difference is due to a proper semantic contribution given by the definite article. It seems that Frege's remark is intended only as stressing the point (ignored by Kerry) that definite descriptions must be taken as singular terms, being therefore essentially different from conceptual terms.

The last passage I want to consider relates to the so-called "Paradox of the Concept Horse", i.e., reference to an object instead of a concept in using the expression 'the concept so-and-so', and it is quite helpful in elucidating the kind of role that the definite article has for Frege. It appears in the *"Logik in der Mathematik"* from 1914 (NS, p. 269/PW, pp. 249–50; my emphasis):

It is indeed strictly a mistake to say 'The concept positive number is satisfied', for by saying this I seem to make [Ich mache dadurch] the concept into an object, as the definite article 'the concept' shows [erkennen lässt].<sup>11</sup> It now looks as though 'the concept positive number' were a proper name designating an object and as if the intention were to assert of this object that it is satisfied.

As the passage suggests, the definite article indicates a sort of act on the speaker's part, i.e., the act of referring to an object instead of a concept. (To make a concept into an object, strictly speaking, is an absurdity from Frege's realistic point of view, so what he means is to refer to one entity and not to the other.) The second sentence of this translation contains the term "intention", which does not really occur in the German text, but Frege formulates it entirely in the subjunctive mood, suggesting that this is something that I "want" to happen (i.e., 'the concept

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<sup>11</sup> Here again the standard English translation might be misleading. 'Erkennen lässt' is closer in meaning to 'gives a hint' or 'indicates'.

positive number' designating an object). Again, the definite article seems to indicate what my intention as a speaker is.

These passages taken together suggest, I submit, the following interpretation: for Frege, the definite article has no properly semantic contribution; its primary function is to indicate, in the language, the *presupposition* of existence and uniqueness or the *intention* on the speaker's part of capturing one and only one object by means of a certain conceptual content. If this is correct, there is an interesting parallel between placing a definite article in front of a conceptual expression and asserting a complete content. In the same way that the assertion of a thought does not add any content to it (1918/19, p. 61), the definite article does not add any content to a definite description. Assertion is an act on the speaker's part made explicit by the assertion-sign, and the use of the definite article also makes explicit a special act on the speaker's part which we might perhaps call an *individuation claim*. The latter adds nothing to the content of the definite description, but merely gives expression to the speaker's expectation that the descriptive content individuates one and only one object. On the other hand, the predicate *is true* can occur without properly displaying this speech act in a non-scientific context, since it can well be employed in mock assertions, e.g., when an actor in the stage says '*it is true that I am the only son of Zeus*'. Similarly, the use of the definite article in some contexts may occur in mock individuation claims, e.g., when the same actor says 'the blue unicorn just crossed the sky'.

What emerges from these considerations is that the definite article in definite descriptions should probably be interpreted as a marker or indicator of the presupposition (combined with an intention to refer to the object uniquely satisfying the conceptual content). In this respect, the definite article is similar to Frege's judgment stroke: the latter does not affect the content that is asserted, but is simply the mark of a speech act, an attitude on the speaker's part regarding a particular content. And an individuation-claim does not affect the descriptive content of the definite description at all, but simply indicates an expectation on the speaker's part that this content captures one and only one object. I will call *Presupposition-Indicator* the thesis that the definite article in definite descriptions serves primarily to indicate an individuation-claim (but does not necessarily express it semantically). As we saw, the evidence coming from Frege's own remarks gives support to the interpretation that he holds Presupposition-Indicator and gives no support at all to the contrary thesis, i.e., that the definite article has a semantic content for him. All possibilities that we considered as

obvious candidates for the semantic content of the definite article are either explicitly ruled out by him or incompatible with some aspect of his doctrine of sense.

## **5 THE INCONSISTENCY BETWEEN PRESUPPOSITION-INDICATOR AND FREGE'S SEMANTICS**

But if Frege embraces Presupposition-Indicator (and this is the only interpretation that emerges more or less clearly from his remarks), then his semantics is hardly coherent. The first and most serious problem is that, according to (P\*), only senses can saturate unsaturated senses. Only senses can lead to individuation. But, according to Presupposition-Indicator, the definite article does not contribute a sense; it does not affect the descriptive content of the definite description. It only expresses an intention or expectation, in the same way that the assertion sign does not affect the thought, but only expresses endorsement. So, it cannot saturate the sense of conceptual expressions to build a saturated sense of definite descriptions.

Second, there is a problem regarding the specification of the propositional content of a sentence with a definite description as subject. In the case of a quantified sentence like 'every *A* is *B*' or 'some *A* is *B*' we can see how the senses of '*A*' and '*B*' combine to form a thought, namely, they are tied together by the sense of 'every *X* is *Y*' which is a binary second order function. But no such thing can happen in the case of 'the *A* is *B*', since the definite article is merely an indicator of an expectation or attitude: it does not combine with senses to form thoughts. There is simply no explanation of how the sense of '*A*' and '*B*' combine to form a thought here. We seem to be forced to say that entertaining the sense of 'the *F*' (or 'the *F* that is *G* and *H*') is not distinct, as regards the content, from entertaining the sense of '*F*' (or of '*F* and *G* and *H*...'), since nothing corresponds to the definite article to "singularize" the sense of 'the *F*' (or 'the *F* that is *G* and *H*').

Third, this theory is incompatible with Frege's view of quantification. In a number of places (e.g., in NS, p. 274/PW, p. 254) he excludes the possibility that a quantifier might attach directly to a singular term beginning with the definite article (like, e.g., 'the greatest soccer player in history exists'), presumably because the quantifier can only attach to a conceptual term. But the definite article in 'the greatest soccer player in history', according to Presupposition-Indicator, does not add anything to the content of the description; it only marks an expectation or attitude. Hence, there should be no difference in terms of content between 'greatest soccer player in history' and 'the greatest soccer player in history', and



quantification should be allowed in both cases. But, as it seems, the expression resulting from placing a quantifier (e.g., existential) before ‘the greatest soccer player in history’ would not be coherent according to Frege.

Fourth, there is a disharmony between this theory and the spirit of Frege’s doctrine of thoughts. He famously distinguishes between: (i) the grasping of a thought, (ii) the judgment (i.e., the act of recognizing that it is true), and (iii) the assertion (which is the expression of the judgment) (e.g., in 1918–19, p. 62). Moreover, he distinguishes the very existence of the thought from the grasping of it by anyone at any time. The thought exists independently of anyone entertaining it. There is no reason to suppose that Frege would see singular thoughts as an exception to this. But if he holds Presupposition-Indicator, his theory has the consequence that, in order to simply entertain a singular thought (no matter whether we are in the realm of science or in the realm of fiction), our intellect has to play some active role in the very formation of the thought since we must first singularize the conceptual content of the description. This mental act (the individuation claim) is not the same as judging; it comes before judging. Yet the very existence of a singular thought would require an individuation claim (otherwise nothing singularizes a conceptual sense) and, hence, a subject. But it seems unlikely that Frege would accept that a mental act would take part in the very composition of a thought content.

## 6 THE *ERSATZ*-DEFINITE ARTICLE IN GGA I §11

Part of the problem discussed so far disappears for Frege’s formal analogue (which he calls “Ersatz”) of the definite article in his formal system. In GGA I §11 he famously introduces the  $\backslash$ -operator, very much in the same spirit in which Russell introduces the inverted iota notation. He defines it in the following way (where  $a$  is any object):

- $$\backslash a = (i) \Delta \text{ iff } a \text{ is a value-range of a function } F \text{ that yields the value True only to } \Delta$$
- (ii)  $a$  otherwise.

In a more contemporary set theoretical terminology, if  $a$  is an object of the form  $\{\Delta\}$ ,  $\backslash a$  has  $\Delta$  as value, and if  $a$  has any other form,  $\backslash a$  has simply  $a$  as value. Presumably, if  $a$  is the extension of a concept  $F$  we could read  $\backslash a$  as representing ‘the  $F$ . If  $F$  is true of one and only one object, ‘the  $F$  will refer to that object; but if  $F$  is empty, or is true of more than one object, then ‘the  $F$  will refer to the extension

itself. If  $a$  is not an extension of a concept, there is no intuitive reading of  $\lambda a$  in natural language. The  $\lambda$ -operator is not a second-order operator (as the definite article in natural language would presumably be) but it can be combined with the value-range function  $\varepsilon'\Phi(\varepsilon)$  (which takes first-order functions as arguments and has their value-ranges as values) thereby yielding the second-order function  $\lambda\varepsilon'\Phi(\varepsilon)$ . Its value is the unique object satisfying the concept (if the argument is a concept under which a unique object falls) or the value-range itself (in all other cases, i.e., if there is no such unique object, or if the function taken as argument is not a concept). So, most likely Frege's intention was to treat the whole complex  $\lambda\varepsilon'\Phi(\varepsilon)$  as the formal *Ersatz* of the ordinary definite article. It has different semantic properties from those suggested by Presupposition-Indicator. The  $\lambda$ -operator (and hence the  $\lambda\varepsilon'\Phi(\varepsilon)$  function) has both a sense and a reference (as Frege claims and attempts to prove in GGA I §29–31), and, hence, does contribute to the sense and reference of a definite description.

Is this a solution to the problem that we have been discussing? The answer depends on how we understand the role of the *Ersatz* and its relation to the ordinary definite article. The latter is part of ordinary language which is, for many purposes, defective, and the defects can be avoided in a formal language by means of technicalities that do not necessarily bear resemblance to the original expressions. But do we have to say that the introduction of the technicalities in Frege's formalism *solves* the problems and tensions present in ordinary language, or do they just *avoid* them? Frege himself is not completely clear about this. The *Ersatz* is highly artificial and only remotely resembles the definite article. As said before, it has no intuitive reading when taking as argument a function that is not a concept (at least Frege does not bother to explain such a reading). And because the explanation of its value for each argument involves the existence and uniqueness of objects satisfying the concepts, there is a fundamental difference between it and the definite article: as we saw, sentences containing the definite article do not have, for Frege, an existential or uniqueness clause as part of their senses. Moreover, the  $\lambda$ -operator is a full first-order function (and, therefore,  $\lambda\varepsilon'\Phi(\varepsilon)$  is a full second-order function) while the definite article could, at most, be regarded as a partial function, as Frege himself notices (GGA I §11):

Here, then, we have a substitute for the definite article of language, which serves to form proper names out of concept-words. [...] Here is a logical risk. For if we were to form out of the words 'square-root of 2' the proper name 'the square-root of 2', we would commit a logical error, since this proper name would be, without further stipulation, ambiguous and just for that reason without reference [...] This risk carried by the definite article is now avoided altogether, since ' $\lambda\varepsilon'\Phi(\varepsilon)$ ' always has a reference, whether the function  $\Phi(\xi)$  is not a concept, or a concept under which more

than one or no object falls, or whether it is a concept under which one and only one object falls.

The “logical risk” is something always present in any use of the definite article (being perhaps essential to it), while it does not exist at all for the  $\lambda$ -operator.

Anyway, this formal *Ersatz* for the definite article never seems to play a role in Frege’s broader views outside his formal system.<sup>12</sup> For example, his remark (already quoted) on the definite article corresponding to a presupposition (and not to a total function) in “*Über Schoenflies: Die logischen Paradoxien der Mengenlehre*” (NS, p. 194/PW, p. 179) occurs in 1906, after the publication of GGA I (1893) and the formal alternative of the *Ersatz*. Hence, he seems to have retained the view of the definite article as a marker even after introducing the artificial definite article in his formal system.

## 7 DEFINITE DESCRIPTIONS: DEVICES OF SINGULAR REFERENCE OR QUANTIFIERS?

More recently there has been an influential line of thought among some philosophers and linguists (e.g., Barwise and Cooper (1981), Evans (1982), and Neale (1990), among others) with an entirely different approach to definite descriptions. This approach breaks with the view that they are genuinely referential devices, and treats them as quantifier phrases instead,<sup>13</sup> i.e., as particular cases of expressions resulting from the combination of determiners (‘all’, ‘some’, ‘most’, ‘few’, etc.) with conceptual expressions (‘men’, ‘student’, ‘King of France’, etc.) or, more generally, open formulas. Barwise and Cooper (1981) famously call expressions of this kind *generalized quantifiers*. A definite description can be treated as a particular kind of generalized quantifier; the definite article is, according to this approach, a determiner, along with ‘all’, ‘some’, ‘most’, ‘few’, etc. A variation of this approach (advocated by Evans (1982, chap. 3)) is to treat the definite article as a binary quantifier, i.e., as a second-order function that has two concepts as argument and associates a truth-value with them. (‘The  $A$  is  $B$ ’ can be treated as ‘the( $A, B$ )’, i.e., as the application of the quantifier ‘the’ to the conceptual expressions ‘ $A$ ’ and ‘ $B$ ’: ‘the ( $A, B$ )’ is true iff there is one and only one  $A$  and it is  $B$ .)

<sup>12</sup> Frege clearly introduces the  $\lambda$ -operator in such a way that it can be placed in front of any argument (object) whatsoever, no matter whether there is one and only one object having a certain property (i.e., the intended reference of the complex formed by the operator and an extension). So, again, Frege’s  $\lambda$ -operator is fundamentally different from Hilbert and Bernays’ (1934, §8) definite article, since no proof is required for its use.

<sup>13</sup> I do not mean the analysis of sentences containing definite descriptions as quantified sentences, as Russell classically proposes; I mean treating definite descriptions as being themselves quantifiers.

This is not the place to discuss the advantages of this perspective (in its several variations) according to its proponents; I simply want to point out that this theoretical alternative was clearly available to Frege. He could easily and naturally have extended his own account of quantifiers as second-order functions to the definite article (provided he abandoned the idea that existence and uniqueness are just presuppositions). The latter could refer to a binary function that takes two concepts as arguments, in the same way that he himself treats ‘Every  $A$  is  $B$ ’ as having the form ‘Every ( $A$ ,  $B$ )’ (i.e., the binary second-order function corresponding to the universal quantifier applied to the concepts  $A$  and  $B$ ), and ‘Some  $A$  is  $B$ ’ as having the form ‘Some ( $A$ ,  $B$ )’, (i.e., the second-order function corresponding to the existential quantifier applied to  $A$  and  $B$ ). On some few occasions Frege contrasts the combination of concept words with the definite article (or demonstrative expressions) with the combination of concept words with indefinite articles, universal or existential quantifiers, but only to stress a categorical gap between the resulting expressions (e.g., in NS, p. 230/PW, p. 213; WB, p. 150/PMC, P. 92). No fundamental reason is presented for his insistence on this asymmetry. Apparently, he wants to retain by all means the referential function of definite descriptions, and this is *prima facie* incompatible with the quantificational approach. (As we saw, according to one interpretation, definite descriptions are for him the primary way of referring to objects, so if they are not referential devices after all, then there is simply no singular reference at all.)

## 8 CONCLUSION

As we saw, Frege holds all of the following theses (most of them quite explicitly and with full generality), including Principles (P) and (P\*):

- (i) Definite descriptions are singular referential devices;
- (ii) The sense of a proper name determines its reference (if there is one);
- (iii) The senses of proper names are saturated and the senses of conceptual expressions are unsaturated;
- (iv) The definite article in a definite description adds nothing to its sense;
- (P) A complex sense is composed only of other (possibly more basic) senses;
- (P\*) Only a sense can saturate an unsaturated sense.

The resulting doctrine is, as I tried to argue, not coherent, and leaves unexplained the sense of definite descriptions. As noticed before, it is a matter of controversy among scholars whether Frege holds the additional thesis

(v) Proper names express the same sense as definite descriptions.

If we interpret him as holding (v) with full generality, then we have to conclude that he has no coherent explanation of singular senses in general (and, hence, no coherent explanation of singular thoughts and, ultimately, of senses). But even if we do not read him as holding (v) with full generality, we have to conclude that his view on definite descriptions is not coherent. This is very damaging for a certain tradition of interpreting Frege in the style suggested by Strawson's (1950) theory. For this interpretation cannot explain the transformation of a conceptual content (unsaturated sense) into a singular content (saturated sense) by means of the semantic contribution of the definite article, since it assumes that the definite article is a mere presupposition-indicator. The leap from descriptive content or conceptual sense to singular content or singular sense is thus not explained except by the speaker's abandoning the descriptive content and just having a particular object in mind (in Donnellan's sense). Hence the Strawsonian interpretation is untenable since it is incompatible with the rest of Frege's semantics. Moreover, for those expressions that are not equivalent with definite descriptions, there seems to be no explanation at all of the singularity of their senses except the very presence of the reference. Hence, as it seems, Frege would have to surrender to some form of Russellianism.

It is surprising that philosophers of language both from the descriptivist and from the referentialist side, who either endorse or criticize Frege's notion of singular sense, seem to overlook this problem. And the problem arises quite apart from the exegetical question of whether he reduces all forms of proper names to definite descriptions. Most authors just mention the fact that senses are responsible for the difference in cognitive value between identities like 'Hesperus = Hesperus' and 'Hesperus = Phosphorus', and therefore the sense of 'Hesperus' and 'Phosphorus' must be different. But we hardly find a more substantial explanation of the nature of the sense of proper names or an explanation of the structure of the sense of definite descriptions themselves. If my discussion in this paper is correct, there is unfortunately no explanation consistent with everything Frege holds.

## ACKNOWLEDGEMENTS

I am indebted to Gottfried Gabriel, Nathan Salmon, Eros Corazza, François Recanati, Ed Zalta, Dirk Greimann, Ludovic Soutif, Ernesto Perini-Santos, Filipe Martone, Matheus Valente, and to an anonymous referee for several suggestions made on earlier drafts of this paper. Research for this paper was supported by FAPESP (Grant 2018/17011-9) and by CNPq (Grant 301317/2017-8).

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