Logic of induction: a dead horse?
some thoughts on the logical foundations of probability

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Resumo: São dois os propósitos deste artigo. Primeiro desejamos examinar porque o projeto de Carnap de construir uma lógica indutiva não foi bem sucedido. De forma a realizar isso, nos apoiaremos na distinção entre o problema da justificação da indução e o problema da descrição da indução. Tentaremos mostrar que a principal razão pela qual o projeto de Carnap falhou foi sua relação com o problema da justificação da indução. Nosso segundo objetivo é propor algumas idéias de como seria um lógica da indução que propositadamente evite o problema da justificação e possa consequentemente ser chamada de uma lógica puramente descritiva da indução. Utilizaremos para isso um conceito de probabilidade presente no *Logical Foudations of Probability* de Carnap chamada por ele de probabilidade pragmática.
Palavras-chave: Carnap, Indução, Probabilidade pragmática, Problema da descrição da indução

Abstract: Our purpose in this paper is twofold. The first is to understand why Carnap´s project of building a logic of induction as a whole was not successful. In order to achieve that we shall make use of the important distinction between the *problem of justification* of induction and the *problem of description* of induction. We shall try to show that the main reason why Carnap´s project failed was its connection with the problem of justification of induction. As a secondary purpose, we want to advance some ideas on how a logic of induction which deliberately avoid the problem of justification and therefore could be called a purely descriptive logic of induction would look like. In order to do that we shall make use of a concept of probability contained in Carnap´s Logical Foundations of Probability called by him the pragmatical notion of probability.
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1 Introduction

For the last 15 years or so, it has been commonplace among philosophers to consider the whole project of building a logic of

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induction as conceived by Rudolf Carnap as fundamentally misleading. In a paper entitled “Why There Can’t be a Logic of Induction,” Stuart Glennan for example compares such project to a dead horse[1]:

> Carnap’s attempt to develop an inductive logic has been criticized on a variety of grounds, and … I think it is fair to say that the consensus is that the approach as a whole cannot succeed. In writing a paper on problems with inductive logic … I might therefore be accused of beating a dead horse.

A similar statement is found in the entry for “Inductive Logic” in J. Pfeifer’s *Philosophy of Science: An Encyclopedia*, written by Branden Fitelsen[2]:

> Moreover, … there are further (and some say deeper) problems with Carnapian … approaches to logical probability, if they are to be applied to inductive inference generally. The consensus now seems to be that the Carnapian project of characterizing an adequate logical theory of probability is (by his own standards and lights) not very promising.

Our purpose in this paper is twofold. The first one is to understand the rationale behind theses claims and why Carnap´s project of building a logic of induction as a whole was not successful. In order to achieve that we shall make use of the important distinction between the *problem of justification* of induction and the *problem of description* of induction. We shall try to show that the main reason why Carnap´s project failed was its connection with the problem of justification of induction. As a secondary purpose, we want to advance some ideas on how a logic of induction which deliberately avoid the problem of justification and therefore could be called a purely descriptive logic of induction would look like. In order to do that we shall make use of a concept

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[1] Glennan (1994), p. 78.
[2] Fitelsen (2006), p. 9.

of probability contained in Carnap´s Logical Foundations of Probability called by him the pragmatical notion of probability[3].

The structure of the paper is as follows. In the Sections 2 and 3, after briefly surveying the main conceptions of induction, we analyze the main features of the logical conception of induction associated with Carnap's school. In Sections 4 and 5 we explore some important relationships between induction and probability. In Section 6 we examine Carnap's system in order to illustrate what we have said in the previous sections and better understand claims such as the ones quoted in this introduction. Finally, in Section 7, we advance some ideas about how a logic which avoids Carnap's justificatory flaws would look like.

2 From inductive generalization to ampliative inferences

The most traditional use of the term "induction" is that which equates induction with what today is known as *inductive generalization* or inference from the particular to the general. Taking a widely used example, if we observe, let us say, 100 ravens and notice that all of them are black, we may generalize that and conclude that all ravens are black. This act of inferring a general statement from particular instances is the first important feature of this traditional meaning of induction. The other is the *purpose* associated with this kind of reasoning. Induction in this sense is conceived as a way of discovering or generating hypotheses, laws or principles; or, broadly speaking, as a sort of logic of discovery.

This use of "induction" has been first taken by Aristotle (at least was him who first used a specific technical term – *epagôgê* – to refer to this inferential process[4]), to whom scientific knowledge is obtained by demonstration from indemonstrable first principles, and knowledge of these first principles is in turn obtained by induction. It is important to remark however that to Aristotle the

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[3] Here we shall follow Carnap and use the adjective "pragmatical" instead of "pragmatic".

[4] The term "induction" comes from Cicero, who introduced the word *inductio* as an exact equivalent for *epagôgê*.

generalization resultant from an induction is not necessarily of an empirical character. In the words of J. R. Milton[5]:

> Among the truths which Aristotle describes as being reached by induction … What we do *not* find are what we are accustomed to think of as empirical generalizations. Aristotle uses the word *epagôgê* and its derivatives over fifty times in his various writings, and the *only* example of a proposition derived by *epagôgê w*hich could reasonably be described as an empirical generalization is the discussion example of all bileless animals being long-lived which appears in *Prior Analytics,* II.23.

Another important conception of induction is the so-called *singular predictive induction,* or the non-demonstrative inference from the particular to the particular. Taking again our raven example, rather than concluding that all ravens are black, in a singular predictive induction we would conclude that the next raven to be observed will also be black. Despite the obvious differences between this meaning and the first one, singular predictive induction can be very fairly taken as a particular case of inductive generalization. We will call this conception of induction understood as inductive generalization and/or singular predictive induction the *classical conception of induction.*

The shift to what we call the *modern conception of induction* took place in the seventieth century with Francis Bacon's *Novum Organum*. While induction in this new sense remained chiefly conceived as generalization from particulars and as a method of discovery, it started to be taken (as explicitly suggested by Bacon) as the chief method (of discovery) of the newly born natural sciences. Accordingly, all aspects of inductive reasoning, in special its conclusions, were taken as being *empirical* in essence. In this way, we arrive at the modern idea (still in vogue today) according to which all science starts from observation and then slowly and cautiously proceeds to theories, which consist basically of generalizations of such observations.

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[5] Milton (1987), p. 53.

Another very important part of Bacon's philosophy of science is that he considered pure inductive generalization as a "puerile thing," incapable *per se* of generating any kind of knowledge. In order to generate authentic scientific knowledge, it has to be supplemented with some additional method, in Bacon's case a method of exclusion intended to obtain the right conclusion. As he puts it[6]:

> But the greatest change I introduce is in the form itself of induction and the judgment made thereby. For the induction of which the logicians speak, which proceeds by simple enumeration, is a puerile thing; concludes at hazard … Now what the sciences stand in need of is a form of induction which shall analyze experience and take it to pieces, and by the process of exclusion and rejection lead to an inevitable conclusion[7]

This heuristic aspect of the modern conception of induction, along with its emphasis on the empirical character of premises and conclusions, is what mostly distinguishes it from the classical conception. However, as mentioned in a previous paragraph, they still share some very fundamental features. First of all, induction in both senses is primarily conceived as a method of discovery (be it of particulars or of general principles). In other words, the role of induction in the scientific enterprise is to produce *new* pieces of scientific knowledge. Another similarity is that both the classical and the modern conceptions can be classified as *structuralist* conceptions of induction, that is to say, the classification of a given reasoning as inductive is based primarily on the analysis of its syntactical structure (whether its goes from particulars to general, whether it makes use of such and such heuristic principle, etc.)

There is still a third common trait between the classical and modern conceptions that, unlike the first two, seems to be a much more essential feature of induction. We are talking about the trivial fact that a conclusion got from an inductive generalization or from a

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[6] Bacon (1620), p. 249.

[7] John Stuart Mill, with his methods of agreement, difference, etc, also made use of the same sort of heuristic principle.

singular predictive induction may be false even though their premises are true. In other words, induction either in the classical sense or in the modern sense is a *non truth-preserving* type of reasoning. The main point of course is that the conclusions of inductive generalizations (with or without some heuristic method of conclusion choice) and singular predictive induction contain information that is not contained in the premises. That I have observed 10.000 black ravens says nothing about the features of the next raven I am going to observe or about all ravens. In these cases, the conclusions go beyond what is stated in the premises; they increase our knowledge. And it is exactly this *ampliative* character of induction what makes it non truth-preserving and also so interesting.

Now, if the distinguishing logical feature of induction is that it is ampliative and consequently non truth-preserving, apart from structural or functional differences, we may say that induction is logically indistinguishable from other types of reasoning, such as abduction for example, which are ampliative too. This viewpoint led some philosophers to extend the meaning of "induction" as to make other ampliative types of reasoning fall under its label.[8] If we go on with this meaning extension we will get to the point of taking induction in a very broad sense and identifying it with the *class of all ampliative* or *non truth-preserving inferences*. That is what we call the *contemporary conception of induction*.

Right away we see that this new conception places induction in sharp contrast to deduction: considering that deduction is truth-preserving and consequently non-ampliative, inductive will then mean non-deductive, and deductive non-inductive. This conception of induction is the one we find in most standard textbooks on logic and induction.

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[8] Charles Pierce, for instance, identifies three types of induction: crude induction, quantitative induction and qualitative induction, where only the first one corresponds to what we have called inductive generalization. See Peirce (1931), p. 756-59.

Now, this contemporary notion of induction embodies very significant changes in relation to the earlier conceptions. Maybe one of the most important is that for the first time induction was explicitly seen as a kind of inference or argument, in contrast to a type of reasoning. To make the difference clear, reasoning is a complex structure that, among other things, may contain arguments, definitions, conclusion choice procedures, etc. In its turn, inference is the very cornerstone of reasoning. In the traditional sense, an inference or argument[9] is a logical relation between a set of propositions and a proposition – the first called premises and the second conclusion – according to which, by its very logical nature, the first entails the second.

Now, if there is such a thing as inductive inference, it should be, due to its very nature, somehow susceptible to a logical analysis. More specifically, by abandoning a simply structural definition and adopting a "logical" one, this contemporary conception of induction placed induction on the same level as deduction and opened the possibility that a logic of induction akin to deductive logic could be developed. As one might expect, these changes brought into scene both those who believe in the existence of such class of inferences and want to develop a logic of induction, as well as those who deny its existence and consequently the possibility of such sort of logic[10].

Another significant change entailed by the contemporary conception of induction is concerned with the alleged purpose of induction. According to the classical and modern conceptions, induction was chiefly conceived as a method of discovery. This was not just a policy on the use of inductive inferences; rather, it was part of the very notion of induction. In its turn, induction as conceived by contemporary philosophers rejected this and any other sort of practical purpose. Despite the historical reasons involved,

[9] Even though the term "argument" may be taken as something similar to "reasoning," we will use it here in the customary way, as a synonymous of inference.

[10] See Fritz (1960), Sellars (1969) and Machina (1984).

this was a direct consequence of taking induction as a sort of argument. If there is some purpose to be fulfilled in the performance of inductive inferences[11] there must be necessarily reference to procedures foreign to the inferential relation itself. Therefore, despite being possibly connected with each other, the purpose in question cannot be taken (with the risk of nullifying the logical conception) as part of the notion of induction. Induction *per se* started to be considered as a purely logical notion.

3 The contemporary notion of induction and the problem of justification

But if induction is the class of all ampliative inferences, then how about fallacies? Are they also to be included in the class of inductive arguments or treated separately? Trivially the first alternative is unacceptable: accepting fallacies as a type of inductive argument is simply to give up the soundness we expect to be present in any inductive argument. Then we are left with two options: to distinguish between good and bad induction or to redefine the notion of induction; in both cases as to take fallacies into consideration. Independently of the path we choose, the basic problem is the same: how to distinguish induction (or good induction) from fallacies.

Surely the most immediate answer would be to invoke the notion of rationality and say that what distinguishes induction (or good induction) from fallacies is that while the first one is in some sense a reasonable, rational inference, the steps from premises to conclusion in a fallacy are unwarranted. Wesley Salmon, for instance, says the following[12]:

> If, however, there is any kind of inference whose premises, although not necessitating the conclusions, do lend it weight, support it, or make it probable, then such inferences possess a certain kind of *logical rectitude*.

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[11] Such as the determination of which hypotheses can be inductively inferred from a given set of evidences.

[12] Salmon (1966), p. 8. The italics are mine.

> It is not deductively valid, but it is important anyway. Inferences possessing it are correct inductive inferences.

So, this alleged "logical rectitude" is what distinguishes (good) inductive inferences from fallacies. But if we just say this we are not saying too much. What precisely is this logical rectitude? What warrants us to classify the inferences that possess it as rational? As one might suspect, this is the famous *problem of justification of induction*, also known as Hume's problem of induction: "How to justify the rationality of inductive arguments?"

The basic difficulty with the problem of justification of induction seems to be that justifying or showing the rationality of an argument is, we fell, tantamount to showing its logical character. Since from a strict point of view there is no logical connection between the premises and conclusion of an inductive inference, we have then a problem that threatens the very foundations of the contemporary conception of induction. In fact, since Aristotle, the problem of justifying the reasonableness of non-deductive arguments has been one of the main sources of suspiciousness against induction. Later on, after Hume's famous critics and the recognition of its importance for the scientific method, the justification of induction has occupied a very crucial place in the philosophy of science. Incidentally, since the publication of Hume's *A Treatise of Human Nature* in 1739 up to now, no satisfactory solution to this problem was proposed[13].

As we have mentioned, even though this problem affects all three conceptions of induction, the effect it has upon each one is not the same. While it may be epistemologically important for the first two conceptions to find a rational justification for the kind of reasoning they are concerned with, the result of such quest does not affect in a decisive way the way they are using to the word "induction." In the case of the contemporary notion the situation is different. As we have seen, in order to properly characterize the

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[13] For an exposition of the kinds of attempts made to solve the problem of justification of induction see Salmon (1966), chapter II.

class of inductive inferences we have to, besides giving a negative criterion (which is of course the argument's being non truth-preserving), also give a positive criterion capable of distinguishing inductive arguments from other arguments which also satisfy the negative criterion (read fallacies.) And independently of our appealing or not to the notions of logical rectitude or rationality, to give such positive criterion amounts to solving Hume's problem.

This simple but at the same time subtle connection between induction and the problem of justification is at the root of the controversy regarding the existence of inductive arguments and the tenability of the project of building a logic of induction. From a philosophical point of view, the whole thing has to do with the very way we are trying to define the class of inductive inferences, that is to say, *intensionally*. Since we want to give a general criterion to say whether or not a given inference is inductive, we will have inevitably to deal with the problem of justifying why such and such inference is in fact inductive. And since one of the distinguishing features of induction will inevitably be the property of reasonableness or rationality (even if under another label), our criterion will have to give an answer to the question of why such and such inference is rational. Because of that, we say that this contemporary conception of induction is or embodies a sort of *intensional* or *justificatory approach to induction*.

Given all this, it is reasonable to wonder if there is not some other way of dealing with induction which does not require such sort of justification endeavor. To start with, independently of finding a necessary and sufficient criterion of inductiveness, there is always a class of ampliative inferences that in a particular period of time is used in a certain category of practical situations and accepted as sound by a certain community of people. So, one possible alternative is to take induction as *this set* of accepted ampliative arguments. We will call this approach to the contemporary meaning of induction the *extensional* or *pragmatical approach to induction.* Despite the obvious objections one may rise against this approach *if taken* as a definition of induction, if we decide to follow this path, our basic concern shall be reduced to the problem of describing the accepted patterns of inductive inference, or, in other words, the so-called *problem of description of induction*.

Despite the fact that only recently philosophers have paid more attention to the importance of the distinction between a problem of justification of induction and a problem of description of induction[14], references to this twofold division can already be found in the golden days of inductive logic:

> The problem of induction … has stimulated two different but complementary types of research. First of all there is the problem of how one can justify the inductive inferences that we do as a matter of fact make, a problem whose solution is seems impossible since the days of Hume. The other approach is that of Bacon, Mill, and Laplace, who analyse the way we make inductive inferences. They try to find reasonable methods of inference, without necessarily giving justification that would go counter to Hume's argument. [15]

It is interesting to observe that according to some philosophers who do not believe in the existence of a (logical) distinction between deductive and inductive arguments, those who believe in it have established such distinction not on logical grounds, but on pragmatic and epistemic ones. Kenton Machina, for instance, says: "As remarkable as it may seem, common attempts to make the primary distinction between inductive and deductive arguments have turned out to generate a pragmatic or epistemic distinction, not a logical one."[16] Later on he adds: "Perhaps, then, the following suggestion will meet with some acceptance: Recognize that the general purpose, all-embracing distinction between deductive and inductive argument belongs to epistemology and rhetoric, not logic."[17]

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[14] Lipton (1991), for instance.
[15] Kemeny (1963), p. 711.
[16] Machina (1984), p. 577.
[17] Ibid. p. 578.

4 The logical notion of probability

Now, if we identify induction with the class of all (rational) ampliative arguments, what can we say about the conclusion of such inferences in the case where the premises are true? This question is pertinent because if we take induction as a kind of inference we will expect to *infer* something from the truthfulness of the premises. However, by definition, even when its premises are true it is possible for an inductive inference to have a false conclusion. Therefore, truth does not follow from truth in this sort of inference. But then what can we conclude about the hypothesis of an inductive inference when its evidences or premises are true? Before answering this question, we will have to talk about a very important aspect of contemporary philosophy of induction without which any presentation of the subject would be incomplete: the concept of *probability*.

If there is something consensual about induction in the philosophical literature is its connection with probability. To most contemporary authors, the philosophy of induction is essentially the same as the philosophy of probability. Even though this association of induction with probability is not new, it was only in the twentieth century that philosophers explicitly took the philosophical analysis of induction as being for all intents and purposes the same as the investigation of the concept of probability. In the preface to the first edition of his *Logical Foundations of Probability*, Rudolf Carnap expresses this view in a very explicit way: “The theory here presented is characterized by the following *basic* principles: (1) all inductive reasoning, in a wide sense of nondeductive or nondemonstrative reasoning, is reasoning in terms of probability.[18]

It will be useful to name this probability concept applied to (or identified with) induction *inductive probability*. This is necessary because while inductive has almost invariably been taken as probable, the inverse does not hold. The twentieth century saw a remarkable proliferation of different ways of saying what

[18] Carnap (1950), p. v. Italics in the original.

probability is[19], and many of these so-called interpretations of probability are not concerned at all with induction, in any sense of the word.

There is nonetheless one school of probability that has explicitly and beyond any controversy taken probability as the key concept in the philosophical investigation of induction. It is the so-called *logical school of probability*. This school has basically taken induction according to what we have named the contemporary conception of induction.

From the point of view of the systematization of principles, Carnap's masterpiece, *Logical Foundations of Probability*, of 1950, represented the great turning point in the contemporary conception of induction. There for the first time, a concise and comprehensive attempt to build a formal system of inductive logic along with a philosophical analysis of both concepts of probability and induction was presented. Carnap's project started in the 1940s and was further developed by Carnap himself and others such as John Kemeny, Richard Jeffrey and Jaakko Hintikka between the 1950s and 1970s. Others such as Carl Hempel, even though not directly working on Carnap's systems, have followed the same approach in their treatments of induction. Before Carnap, others such as John Keynes, Harold Jeffreys and B. Koopman have given the same inductive treatment to probability.

But how precisely does this concept of probability fit into induction? To any person with a little inclination to philosophical thinking the answer will be straightforward. If we reason in terms of certainty and necessity, we may say that a deductive inference is that in which the truth-relation between premises and conclusion is a certain or necessary one: the truthfulness of the conclusion necessarily follows from the truthfulness of the premises. On the other hand, since in an inductive inference the conclusion may be false even when the premises are true, this truth-relation is not certain, but just *probable*: in the case the premises are true, it is just

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[19] For a description of the several interpretations of probability see Weatherford (1982).

probable, rather than necessary, that the conclusion is also true. If we follow this approach, we will say that inductive inference is the same as *probable inference*; and the sort of conclusion produced by it in the case where the premises are certain is of a probabilistic nature.

However simple this reasoning may appear to be, we should be careful not to overlook the fact that it involves two different and independent positions concerning probability and induction. While the first one makes reference to a relation between two propositions, the other talks about the status of one of these propositions when the other is known to be true. To make sure that there is really a difference, consider a language where we have certain and probable statements. It is quite reasonable to suppose that if *h* is certain, *h* is probable. By making use of this inferential schema we will have conclusions marked with a probability modal operator obtained through an inference that itself is not probable, but truth-preserving: whenever *h* is certain, *h* will be probable. On the other hand, one may suppose that *e* and *h* are inductively related to each other in such a way that *e* gives inductive support to *h*, but nevertheless *h*'s truthfulness has nothing to do with *e*'s probability. In this case, what is of interest here is an inductive or probable relation concerning the truthfulness of two propositions, which may have nothing to do with other qualities propositions may have. We will call these two positions, respectively, the *status approach* to inductive probability and the *relation approach* to inductive probability.

This second, relational way of understanding inductive probability was the one taken by Carnap's logical school. In addition to conceiving induction in relation to probability, Carnap explicitly identified it as a logical relation of *evidential support* between two propositions, one named hypothesis and the other evidence. While the relation of logical deduction establishes a necessary connection between premises and conclusion, the relation of inductive support establishes just a confirmatory or probable connection between evidences and hypothesis.

To Carnap, the confirmation conferred by a piece of evidence to a hypothesis is a purely semantical relation independent of any kind of empirical consideration, be it one's opinion or the relative frequency with which hypotheses of the same kind have occurred in connection with similar evidences. In other words, it is a completely *objective* or *logical* notion. The following quotation illustrates well these points: "Deductive logic may be regarded as the theory of the relation of logical consequence, and inductive logic as the theory of another concept which is likewise objective and logical, viz. probability$_1$ or degree of confirmation."[20] As one might suspect, this conception is essentially the same as the one we have called in Section 2 the contemporary notion of induction.

Coming back to the relation and status approaches to inductive probability, this distinction is particularly important because the place one puts the notion of probability in his analysis of induction will determine several foundational aspects of his philosophy of induction. In particular, it will allow one to give or not an answer to the question we have posed at the beginning of this section.

Clearly, if we adopt an exclusively relational approach, it will be somehow difficult to give any kind of answer to our question. In fact, most philosophers who have taken this position defended that, in an inductive inference, from true premises we cannot infer anything whatsoever. To logical probabilists, probability is exclusively a logical relation between propositions akin to the relation of logical deduction. It is not a property of propositions. Consequently, propositions are not probable *per se*, but only in relation with some body of evidence. This of course has

[20] Carnap (1950), p. 43. The second name given to this logical concept of probability – *degree of confirmation* – is of special significance to us. As the word "degree" indicates, such conception of probability is intent to be an essentially numerical one. This has to do with Carnap's threefold division of probability concepts. According to him, there are three sorts of logical concepts of confirmation: the *qualitative* (positive or classificatory), the *comparative* and the *quantitative* (or metrical) concepts of confirmation.

implications to the very definition of induction as a kind of inference. Carnap is very clear about that[21]:

> If we wish to use the word ‘inference’ … we may say that the hypothesis *h* is inductively inferred from the evidence *e*. … But in this case we must be careful not to overlook the fact that the probability value characterizes not the hypotheses … but rather the inference from the evidence to the hypothesis or, more correctly speaking, the logical relation holding between the evidence and the hypothesis … Thus we see that from the evidence *e* together with the statement ‘the probability of h with respect to e is 1/5’ we can infer … neither *h* itself, which may be false, nor a statement of the probability of *h*, which would be meaningless. In fact, nothing can be inferred from those two premises.

That position is, for obvious reasons, unsatisfactory. In practical situations, we want to be able not only to assert that *e* gives such and such inductive support to *h* but also, in appropriate cases where *e* is true, to detach *h* from *e* and conclude something about it. For instance, it may happen that a theory or hypothesis has to be highly confirmed before it can be cited as the explanation of anything, or juries have to bring in an unconditional verdict “Guilty” (or highly probably guilty) before the accused can be sentenced. However, according to traditional logical probabilist’s view, none of that could be done.

This problem became known among philosophers as the problem of *inductive detachment*, i.e., how to detach the (probability qualified) conclusion of an inductive inference from its premises. Trivially, solving this problem means to go from a relation approach to a status one.

Another aspect that will be determined by the position we chose is related to the problem of justification. If we do like the logical probabilists and decide to take induction (or probability) as being an *objective* or *logical* relation between propositions, we will

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[21] Carnap (1950), p. 33. In this and other statements by Carnap to be quoted in this section reference will be made to a numerical value characterizing the inductive relation between hypothesis and evidences. That is due to already mentioned quantitative aspect of Carnap’s approach.

have to show that there is effectively such kind of relation between the propositions we believe are inductively connected to each other. From an analytical point of view, this implies having to disclose the internal structure of the relation and showing it to depend solely on *a priori* principles. In other words, we will have to show (and justify) that the structure by itself, without any external help, can tell us whether or not (and to what extent) one proposition supports other proposition. Adopting a relational position brings inevitably the necessity of dealing with the problem of justification. Because of that, we can claim the logical school's position to be essentially in accordance with what we have named the justificatory approach to induction.

5 The pragmatical notion of probability

These two aspects, the inability to infer anything from inductive inferences and the necessity of dealing with the problem of justification, are the two main (bad) consequences of adopting the first position. But how about the second one? Is the status approach somehow incompatible with the first position? It will be free from the two mentioned problems? To start with, clearly it is not, in any sense, incompatible with the decision of taking probability as a relation between propositions. In fact, it seems to us that the most natural way of dealing with the problem is to consider both the inference itself and its conclusion as probable.

Carnap has already pointed out something very similar to that. While most of the time being very strict about the possibility of inductively inferring something from the truth of an inductive premise, Carnap has given some few hints about how sometimes that movement may after all be possible. For instance, talking about what he called the *methodology of induction*, he says that "If *e* expresses the total knowledge of [an agent] X at the time *t*, that is to say, his total knowledge of the results of his observations, them X is justified at this time to believe *h* to the degree *r* […]"[22] Elsewhere

[22] Carnap (1950), p. 211.

he says: “If *e and nothing else* is known by X at *t*, then *h* is confirmed by X at *t* to the degree 2/3.”[23] In other words, if the mentioned conditions are satisfied, *h* can be taken as a confirmed or probable hypothesis. Then, should we conclude that Carnap is contradicting himself when he says that nothing can be inferred from an inductive inference? Not quite so. Right after the above statement he adds[24]:

> Here, the term ‘confirmed’ does not mean the logical (semantical) concept of degree of confirmation … but a corresponding *pragmatical concept*; the latter is, however, not identical with the concept of degree of (actual) belief but means rather the degree of belief justified by the observational knowledge of X at *t*.

So, we have here a clear distinction between a logical, on the one hand, and a pragmatical concept of probability on the other. This pragmatical concept is an instance of what we have called the status approach to inductive probability. Of course, Carnap is here talking about a quantitative concept akin to his degree of confirmation. However, given his previously explained distinction between the qualitative, comparative and quantitative notions of (logical) confirmation, we may fairly suppose that, in addition to what he calls degree of justified belief, there is also a comparative and qualitative pragmatical notion of probability. In what follows, we will make use of the term “pragmatical probability” in a broader, not necessarily quantitative sense.

According to Carnap, the point where the logical and the pragmatical concepts of probability interact is at the application of inductive logic, conceived exclusively as the logic of the relation of inductive support. As soon as we have such a logic, we can, provided the evidences are known and certain restrictions are satisfied, conclude that the hypothesis at hand is (pragmatically) probable. These restrictions have to do with the expression “and

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[23] Carnap (1946), p. 594. Italics in the original.
[24] Ibid. The italics are mine.

nothing else is known" in the quotation above and have been taken into account in Carnap's philosophy by what he called the *requirement of total evidence*[25]. Briefly put, the requirement of total evidence states that in order to apply inductive logic to, for instance, get thementioned pragmatical probability, one must make sure that the evidences represent all the available knowledge. This is of course needed because *e* may be an evidence for *h* when taken in isolation, but against or neutral to it when taken in conjunction with *e'*. In the rest of this paper we will refer to such sort of restriction as *total evidence conditions*.

Another important point contained in the quotation above is the reference to belief. According to Carnap, even though this pragmatical concept is not "identical with the concept of degree of (actual) belief," it is still a sort of belief, namely that which is "justified by the observational knowledge of X at *t*." Others like Keynes have made similar points about the connection between logical probability, belief and justified belief (or pragmatical probability): "The theory of probability is logical, therefore, because it is concerned with the degree of belief which is *rational* to entertain in given conditions, and not merely with the actual beliefs of particular individuals, which may or may not be rational."[26]

From this we can lay down two important features of this pragmatical concept of probability. First, it is a sort of belief and, therefore, not a logical, but an *epistemological* notion. For that reason, we will also refer to this new concept as the *epistemic concept of probability*. Second, it is not, properly speaking, the same as beliefs people ordinarily have. Rather, it is that kind of belief which is obtained in a justified or rational way. More specifically,

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[25] Carnap (1950), p. 211-13.

[26] Keynes (1921), p. 4. Because of passages like that, some authors interpret Keynes conception of probability as being essentially epistemic, and not logical. See for instance Fitelsen (2006). As far as we are concerned, we take the traditional interpretation according to which even though Keynes' use of some terms may not be as clear and uniform as Carnap's, his main concern is with a logical concept of probability.

the belief in *h* is rational if and only if there is a proposition *e* such that *e* is certain, *e* gives (logical) evidential support to *h*, and *e* expresses the total available knowledge. Therefore, the pragmatical probability as conceived by logical probabilists is essentially dependent on the logical one. On the other hand, it is in the formation of these rational degrees of belief that the logic of induction finds its more important application.

Inside Carnap's tradition (but not precisely inside Carnap's works), much has been talked about this pragmatical notion of probability. Despite "accidental" references like the ones we have quoted, this notion has been extensively discussed in connection with the problem of inductive detachment. As we have mentioned, due to the necessity of getting something inferred from inductive inferences, many philosophers felt compelled to deal with a status approach. The idea was that the problem of detachment is to be solved by specifying certain conditions according to which the conclusion of an inductive inference could be detached from the premises and taken as *accepted*.[27]

Regarding the second question, whether the status approach will be free from the two mentioned problems, we believe the answer is 'yes.' The first problem, not to allow anything to be concluded from an inductive inference when its premises are true, is trivially solved. After all, the notion of pragmatical probability is defined as that status the conclusion of an inductive inference gets when its premises are known to be true and some total evidence conditions are satisfied[28].

[27] See Kyburg (1964), Hintikka & Hilpinen (1966) and Lehrer (1970).

[28] An objection one may raise against this conclusion is that while our problem concerns inferring something when the premises are true, the pragmatical probability as defined by Carnap can be applied just in those cases where the premises are *known to be true*. A foundational reply to this would say that induction per se, along with all concepts related to it (such as the notion of probability), is itself an *epistemic* notion. As such, the correct definition of induction would be one that makes reference not to truth, but to knowledge of truth. In this way, our problem should be restated as "what can we say about the conclusion of inductive inferences in the case where the premises *known* to be

About the second problem, the necessity of dealing with the problem of justification, there are two points to be considered. First, since the status approach is not committed to the inductive relation that allows one to classify a hypothesis as probable but just to the status itself, we will not be forced to say why the step from evidences to hypothesis is rational. However, and this is our second point, as we have seen, to Carnap the notion of pragmatical probability is dependent on the relation of inductive confirmation: if *e* gives evidential support to *h*, *e* is known to be true and expresses the total of our knowledge, then *h* is pragmatically probable. It is just because of this connection that we can classify these beliefs (or degrees of beliefs) as rational. Therefore, if we equate epistemic probability with rational (degree of) belief in the way Carnap does we will fall again into justificatory matters.

A possible solution to this is to take inductive and rational in the way we have suggested at the end of Section 3 and adopt a *purely pragmatical or descriptive approach to induction*. According to this approach, what characterizes an inductive argument is it's being accepted as so by a certain community. Whether or not *e* gives evidential support to *h* is not any more a question of logical analysis, but simply a matter of how much the inferential pattern exemplified by the argument <*e*, *h*> is practically accepted. In this approach, the definition of pragmatical probability would remain the same – *h* is pragmatically probable if there is an evidence *e* such that *e* gives evidential support to *h*, *e* is known to be true and expresses the total of our knowledge – only the way we will interpret "*e* gives evidential support to *h*" will be different. Trivially then, our main concern in this representational approach will be the description or representation of inductive patterns of inference, without any concern whatsoever for their justification.

true?" Of course this view of induction as intrinsically epistemic is not new. After all, the so-called classical interpretation of probability takes probability essentially as a measure of our ignorance. Keynes also has taken probability as intrinsically connected with the notion of certainty and belief. See Weatherford (1982).

6 Carnap´s logic of induction

In order to illustrate our claim that Carnap´s project of inductive logic fits into what we have called the justificatory approach to induction it is useful to take a closer look at Carnap´s work. And a good way to start is to look at the features Carnap attributed to the relation of inductive support that is supposed to exist between hypothesis and evidence.

In an often quoted passage of his *Logical Foundations*, Carnap writes: "Since we take semantics as the theory of the meanings of expressions in language and specially of sentences …, the relations [between] *h* and *e* to be studied may be characterized as *semantical*."[29] One very common way Carnap used to use to clarify the nature of this semantical relation was to compare inductive logic with deductive logic: "The principal common characteristic of the statements in both fields [deductive and inductive logic] is their independence of the contingency of facts. This characteristic justifies the application of the common term 'logic' to both fields."[30] Elsewhere he details what this independence of contingent facts is supposed to be[31]:

> It seems to me, however, that an elementary statement in inductive logic … expresses a *purely logical relation* between the two sentences involved in the same way that an elementary statement of deductive logic does … The relation is in both cases purely logical in the sense that it depends merely upon the meanings of the sentences.

In accordance to what we have labeled the relation approach to induction, the idea of Carnap's logic of induction was to formalize a purely logical relation of inductive support in the manner as deductive logic formalizes the relation of logical consequence or deductibility. In the same way that by simply giving a semantical structure able to assign meaning to the sentences of a language we automatically set the relation of logical consequence

[29] Carnap (1950), p. 20. The italics are mine.
[30] Carnap (1950), p. 200.
[31] Carnap (1946), p. 596. The italics are mine.

between all these sentences, with a similar endeavor and with no additional non-logical assumption we set a (numerical) relation of confirmation between the sentences.[32]

How Carnap tried to achieve this goal can be seen through a quick look at the system of induction he presented in *Logical Foundations of Probability*. Carnap's initial project was to define a sort of function called by him c-function which when applied to hypothesis *h* and evidence *e* would return the degree of confirmation given to *h* by *e* (in symbols: c(h,e).) In order to achieve the goal described in the above quotations, this function would have to be defined in purely semantic grounds depending "merely upon the meanings of the sentences" *h* and *e*. Clearly enough, this requires that no principle other than purely logical ones should be used in the definition of *c*.

The fundamental concept of Carnap's system of inductive logic is the notion of *state-description*. Given some specific language L_N (where N amounts for the number of individual constants of L), a state-description is a sentence which, by affirming or denying each property of each individual, completely describes a state of the world. From this notion of state-description (which can be fairly thought of as a sort of possible world) we get what he calls *range of a sentence*: If *h* is a sentence of L_N, the range of *h* is the class of all state descriptions in which *h* holds. By defining the *weight* of a sentence *h* (in symbols: m(h)) through these two concepts, we can then characterize the degree of confirmation given to *h* by *e* as the ratio between the weight of $h \wedge e$ and the weight of e:

$$c(h,e) = \frac{m(h \wedge e)}{m(e)}$$

[32] This same idea is found in Hempel's "Studies in the Logic of Confirmation," where he says that the purpose of the logic of confirmation is "to set up purely formal criteria of confirmation in the manner similar to that in which deductive logic provides purely formal criteria for the validity of deductive inferences." Hempel (1945), p. 9.

The central question now is then how to define the weight m(h) of a sentence. The simplest way to do that is to take m(h) as the proportion of possible worlds in which *h* is true or, in other words, the ratio between the number of state-descriptions in the range of *h* and the total number of state-descriptions. This is of course equivalent to assigning to each state-description the weight of 1/(number of state-descriptions) and define m(h) as the sum of the weights of all state-descriptions which belong to the range of *h*. Carnap calls this weight function and the corresponding c-function obtained from it $m^{\dagger}$ and $c^{\dagger}$, respectively. This approach, which Carnap attributes to the early Wittgenstein, is essentially nothing more than the classical definition of probability. The basic difference is that in this case the probability value would be dependent on the language in which the hypothesis and evidences are to be formulated.

The problem that Carnap sees with this $c^{\dagger}$ c-function is that it would not allow us to learn from experience, that is to say, independently of the evidence *e* we take, $c^{\dagger}$(h,e) is always the same. He then proposes a new c-function, c*, that is not plagued by this sort of problem. The distinguishing feature of c* is that it no longer considers all state-descriptions as being equal. Instead, it introduces a definite bias towards uniformity by favoring more homogeneous state-descriptions. To accomplish this, Carnap introduces the notion of *structure-description*: “j is the structure-description corresponding to Z_i (or, Z_i belongs to the structure-description of j) in $L_N =_{df} Z_i$ is a Z in L_N, and j is the disjunction of all Z which are isomorphic to Z_i arranged in lexicographical order.”[33]

Two Z’s are isomorphic if and only if one can be derived from the other by merely exchanging some individuals for others by means of a one-to-one correlation. The idea of c* then is to treat each of these structures as well as the state-descriptions inside them as equiprobable. That is to say, to each structure-description it will be assigned a weight of 1/(number of structure-descriptions) and to

[33] Carnap (1950), p. 116.

each state-description inside a specific structure-description *s* a weight of (weight of *s*) x (1/(number of state-descriptions inside *s*)). The new weight m*(h) of *h* would then be defined as the sum of the weights of all states descriptions in the range of *h*. As usual, c*(h,e) is defined as the ratio of m*(h $\wedge$ e) to m*(e).

Now we are in a position to analyze the claim that c* satisfies the purpose of the logic of induction. To begin with, we may adopt a sort of orthodox position and state that if some system of induction is to be classified as logical, then it must be not only *a* logic of induction but *the* logic of induction. In the context of Carnap's formalism, this means that the c-function which Carnap takes as the basis of his logical system should be arguably a unique and universal way of assigning degrees of confirmation to pairs of hypothesis/evidence sentences (or at least the core of confirmation reasoning which all the other not-so-universal c-functions should be based on.) It is in this direction for example that Glennan argues for the thesis that there can be no logic of induction "in the sense of no uniquely determined *c* function."[34] The example he gives is a situation where $c^{\dagger}$ would be preferred over c*.

It should be noted that in the very development of Carnap's inductive system we find some support for this conclusion. While in *Logical Foundations of Probability* Carnap did present c* as the proper c-function of inductive logic, in later works he no longer argued that one c-function is satisfactory in all cases, but tried rather to develop a theoretical description of an infinite continuum of c-functions called λ-continuum (the parameter λ is supposed to indicate how sensitive the corresponding c-function is to "learning from experience.")[35] And as Carnap (1952) himself concedes, no one value of λ is "better *a priori*" than the others. In Carnap's view then, the inexistence of a unique c-function does not seem to be a strong argument against the possibility of a logic of induction. After

[34] Glennan (1994), p. 82.

[35] Carnap (1952). In more recent works, Carnap has proposed two more additional adjustable parameters γ and η. See Carnap (1980).

all, it may happen that even though c* cannot be shown to be the best c-function, it is, as Carnap wished, a purely logical notion.

In order to appreciate this claim, it is important to note that even though c* may have some advantages over $c^{\dagger}$ in the situations Carnap considers, both of them make use of the same basic principle: the *principle of indifference*. Although in *Logical Foundations of Probability* Carnap denies such dependence and defends that because the mentioned principle "leads sometimes to quite absurd results and in its strongest form even to contradictions, it must be rejected"[36], later he retreated from this and went on to defend that the principle of indifference is in fact to a purely logical assumption[37]:

> ... the statement of equiprobability to which the principle of indifference leads is, like all the other statements of inductive probability, not a factual but a logical statement. If the knowledge of the observer does not favor any of the possible events, then with respect to this knowledge as evidence they *are* equiprobable. The statement assigning equal probabilities in this case does not assert anything about the facts, but merely the logical relations between the given evidence and each of the hypotheses; namely, that these relations are logically alike.

As it would be expected, this point is far from being uncontroversial. In fact, in the same way that the principle of equiprobability has been the most attacked feature of classical systems of probability (as Carnap himself pointed out), it has been one of the most indigestible characteristics of Carnap's inductive logic[38].

Even though we think there are plenty of reasons not to accept Carnap's point that the principle of equiprobability is a logical principle of induction[39], it is not our intent here to engage in

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[36] Carnap (1950), p. 518.
[37] Carnap (1955), p. 22. Italics in the original.
[38] See Weatherford (1982), sections II.11 and III.11, and Salmon (1966), sections V.1 and V.3.
[39] For a couple of arguments against the principle of indifference see Fitelsen (2006).

this sort of debate. Rather, we just want to use this controversy as an example of the claim that the relation approach to induction inevitably brings us to justificatory issues. Given what we have exposed so far, it is quite trivial in which point Carnap gets involved in justificatory issues. Since a semantical notion has to make use of no other principles than purely logical ones, in order to make the point that his concept of degree of confirmation is a logical concept, he has to make sure that all principles his inductive logic is based on are in fact logical. But since one of these principles, the principle of indifference, was not able to form a consensus regarding its logical nature, Carnap had to engage himself in justificatory issues **intent** to show that such principle is in fact a logical one. And exactly because his arguments were not convincing at all, his project as a whole was taken as a fail.

7 Towards a representational logic of induction

At this point one may wonder if what we are have called a purely descriptive approach to induction is a possible enterprise. After all, we have seen that the most influential tradition of inductive logic, which was supposed to be essentially descriptive, was not itself able to keep distance from justificatory issues. And this of course was not due, let us say, to the mathematical resources employed by Carnap and his followers, but in fact to the very idea held by these philosophers of what the logic of induction is supposed to be. Therefore, in order to show that a descriptive approach to induction is a tenable project, we will have to somehow rethink the traditional conception of logic of induction in such a way as to make it susceptible to such a purely descriptive account. By so redefining the purpose of the logic of induction, we will try to show that our dead horse is perhaps not so dead after all.

From a general point of view, the task of the logic of induction as conceived in Carnap’s tradition could be divided into two:

(i) To set a specific way through which probability values are obtained, that is to say, the conditions according to which one statement gives evidential support to another; and
(ii) To lay down the rules according to which probability values are related to each other or, in other words, the logical relations that are supposed to hold between probable statements.

Let us, for the time being, name the parts of the logic of induction responsible for each one of these tasks, respectively, *model of confirmation* and *calculus of confirmation*. Johnathan Cohen defines these two tasks in the context of a numerical approach as follows[40]:

> Two problems in confirmation theory are not always sufficiently distinguished from one another. … On the one hand there is the semantical problem of deciding, in each case, what are the elements of which confirmation-functors are functors and what metric is most appropriate for the assignment of values to these functors. On the other hand there is the syntactical problem of determining any compatibilities or incompatibilities that may hold universally between such assignments. To construct a calculus of confirmation is to solve the latter, not the former.

Right after the above quotation, Cohen correctly classifies the calculus of probability as a calculus of confirmation. Indeed, the only sort of value-determination the calculus of probability does is to get derived probabilities from prior ones: except in limiting cases such as p(h,h) =1, it says nothing about how to assign such prior probability values. This task is responsibility of what we have called model of confirmation. Using the notation of elementary probability theory, we would say that while the purpose of the model of confirmation is to determine, to any pair of sentences *e* and *h*, the probability value P(h,e) of *h* given *e* or, in inductive logic's terminology, the inductive support given by *e* to *h*, the goal of the calculus of confirmation is to establish the rules according to which

[40] Cohen (1966), 463-464.

different probability statements P(h,e) should be related to each other.

Following Carnap, we will now try to establish a sort of parallel between formal deductive logic and inductive logic, as understood according to the above-mentioned division. We first of all note that if we change "probability value" for "true" in the above paragraph, we will get something very similar to the way deductive logic deals with truth-values. What we mean is that in the same way that formal deductive logic gives no sort of effective procedure to decide whether a sentence is true or false (except in limiting cases such as $\alpha \wedge \neg\alpha$) but just sets the logical constraints according to which truth is obtained from truth, the calculus of confirmation also does not say how one sentence confirms another, but just sets the logical cannons which confirmation statements are supposed to satisfy. Not less interesting is the following conclusion: akin to the inferences set by formal classical logic, the inferences set by the calculus of confirmation are, as a quick inspection of the probability calculus will show, deductive rather than inductive. They have the sole purpose of setting the *necessary* and consequently truth-preserving restrictions the reasoning about confirmation is supposed to obey.

The calculus of confirmation being the deductive part of the logic of induction, it is needless to say that the model of confirmation will be its inductive part. In fact, as we have said, it is the goal of the model of confirmation to set down the process by which hypotheses are inductively supported by evidences. With this observation in mind and considering the previous paragraph discussion, we note that deductive logic has no component similar in purpose to inductive logic's model of confirmation. The determination of how to assign truth-values to sentences is completely outside the scope of the theorist who is building his logical system: it belongs to the theory of knowledge rather to logic. This is relevant because if we say that inductive logic is a sort of logic in the sense formal deductive logic is, then we are assuming that a component able to determine the truth-value of sentences could be added to formal deductive logic without changing the meaning logicians and philosophers attribute to "logic," however fuzzy it may be. Clearly enough, hardly any one slightly acquainted with logic will take seriously this assumption. If however, for the sake of argument, we accept such postulation, we will have to accept that logic would get merged into the theory of knowledge. As such, it would have to deal with that component of knowledge which, despite being the most controversial of all, has always been present in one way or another in the epistemological theories: the notion of *justification*.

This point is important because as we have seen, inductive logic does have the above-mentioned component which deductive logic lacks. Therefore, the conclusion we have made regarding the possibility of deductive logic's having added to it a way of getting truth-values applies with the same intensity to inductive logic. In other words, since inductive logic has to somehow determine the degree of confirmation which evidence *e* gives to hypothesis *h*, the component responsible for that, the model of confirmation, could be taken in a very important sense as much more concerned with the theory of knowledge than with logic. As such, it will have inevitably to deal in some way or another with the justificatory issues involved in that field. That this is so can also be seen by recalling that inductive inferences, by being ampliative, bring necessarily new pieces of knowledge which, due to their not being contained in the premises, will require some sort of justification.

The important point for us in all that is that the model of confirmation is, we may say, the window through which the problem of justification of induction comes in the scene. This conclusion is of course anything but surprising: being the only part of inductive logic which deals with inductive inferences, there is no other place the problem of justification of induction could appear except in it. However, from the point of view of our endeavor of conceiving a purely descriptive account of the logic of induction, it is fundamental to know where precisely the problem of justification takes place in order not to take it into account.

It should be observed that the definition of inductive logic's purpose given by our twofold task division does not take into account the task of detaching the hypothesis from the evidences and concluding something like P(h). The reason for that is that the problem of detachment is according to Carnap not concerned with the logic of induction itself but with its application. This is of course a problem if we want a logic of induction primarily designed to deal with the pragmatical notion of probability rather than with the logical notion of probability. At a first glance, it seems we have two basic alternatives: to include one more component to the above mentioned division in such a way as to take into account the mentioned task or to leave it like that and conceive another logic of induction intent to deal with these "detached" plausible hypothesis. Considering what we have just concluded about the model of confirmation and our willingness of having a purely descriptive account of the logic of induction, it is understandable that we should follow the second alternative and try to discover what such new logic of induction should look like.

Given an application of the logic of induction and therefore a set of statements of the form "the degree of inductive support given by *e* to *h* is *x*" or, if we want to stick to a qualitative approach, "*e* inductively supports *h*", our basic problem would be then to formalize the process through which hypothesis *h* is detached from evidence *e*. Since as we have seen this is done when *e* is (known to be) the case and some total evidence conditions are satisfied, sentence "*e* inductively supports *h*" can be seen as a sort of *inductive implication* where the truth of *e*, we may say, inductively implies the plausibility of *h*. From this perspective, *e* may be seen as the antecedent of the inductive implication, *h* as the consequent and the mentioned process of detachment as a MP-like inferential relation stating that (under the condition that some total evidence condition is satisfied) "*h* is pragmatically probable" can be inductively concluded from "*e* inductively implies *h*" and "*e* is the case." Accordingly, we will call the component of our new inductive logic responsible for such inferential process the *relation of inductive consequence*.

Supposing that we have such inferential mechanism at hand, we will need also to reason about the inductively obtained probable statements. That is to say, we will need a logical system able to operate on the deductive level for saying which constraints pragmatically probable statements are subject to. This, we must concede, is already done by what we have called calculus of confirmation. Taking a quantitative approach based on the probability calculus as example, our detached hypotheses will be probability formulae of the form P(h) = x, whose logic is trivially taken into account by the calculus of probability. However, as the name chosen by Cohen indicates, the calculus of confirmation does a bit more than only reasoning about such plausible formulae: it also reasons about sentences of the form "*e* inductively supports *h*" or, what is the same, inductive implications of the form "*e* inductively implies *h*." In the case of the probability calculus, these two tasks are performed by the same system because P(h,e) and P(h) can always be derived from one another. But of course it does not need to be always like that. Therefore, we will separate these two tasks and call the component of the logic of induction responsible for the first the *calculus* or *logic of pragmatical probability* and the component responsible for the second the *calculus* or *logic of inductive implication*.

In addition to these three parts, the logic of induction should obviously also provide a way to represent the inductive implications and the pragmatically probable hypotheses inferred from them. We will name this fourth component the *inductive-probable language*.

Now that we have got a logic of induction with four basic components – the relation of inductive consequence, the logic of plausibility, the logic of inductive implication and the inductive-plausible language – we may wonder if it really has the descriptive purpose our pragmatic approach to induction requires. To start with, we point out that due to its not taking into account the task of saying whether (and to what extent) *e* confirms *h*, our logic of induction will not get involved into the problem of justification of induction. Another consequence of not having nothing akin to the model of confirmation is that the confirmation statements which the logic of inductive implication is supposed to reason about and which the relation of inductive consequence will act upon to “extract” the plausible facts will not be settled by the system, but rather shall come from *outside*. Consequently, rather than being concerned with how facts inductively support others, our logic of induction’s main concern will be how to provide a logical framework where inductive implications along with any inferential capability they may posses could be properly *represented*. In other words, our inductive logic’s purpose will be shifted from the problem of “generating” confirmation statements to the problem of representing or describing them.

At this point it may be useful to recall our previous discussion about inductive logic and deductive logic to conclude that this new sense of inductive logic perhaps deserves much more the title “logic” than its old justification-laden cousin. As it is widely recognized, one of the main purposes of deductive logic is to serve as a logical framework for representing certain sorts of statements and drawing all logical consequences which may be entailed by them. As we have already observed, nothing is said there about whether or not these statements are correct or true. The responsibility of picking true or reasonable statements belongs to the theorist who will use deductive logic, not to deductive logic itself. Similarly, in our logic of induction, now called *descriptive or representational logic of induction*, nothing is said about how hypotheses are confirmed by evidences or whether such and such evidence confirms such and such hypothesis. Its purpose is rather to serve as a framework for representing inductive implications and drawing the plausible hypotheses entailed by them in a specific knowledge situation. The responsibility concerning the rationality of the represented inductive inferences performed inside inductive logic belongs not to inductive logic itself, but to the knowledge engineer who is making use of it.

Now, if our probable-inductive language, along with the inferential mechanism provided by the logic of induction, is able to represent the axioms of a calculus of inductive implication[41] which tell us how to obtain inductive implication statements from inductive implication statements, then it sure will also be able to represent specific ways according to which inductive implication statements are obtained from something else (expressible of course in our probable-inductive language) than inductive implication statements. In other words, it will be able to represent what we have called model of confirmation. In contrast to what one may think, this possibility of representing models of confirmation is in complete accordance with our descriptive approach to the logic of induction. In the same way that, by allowing one to represent what he thinks to be true, deductive logic does not commit itself with the justification of such "true" statements, allowing one to represent the way he thinks inductive statements are "generated" does not commit our inductive logic to the justification of such model of confirmation. The goal of the logic of induction itself is nothing more than to serve as a logical framework where inductive implication axioms of several sorts, including the sort of axioms which could be taken as model of confirmation, can be represented, being the rationality of what these axioms completely outside the scope of the logic. We call the logic of induction so used an *applied logic of induction*.

8 Conclusion

In this article we analyzed what we think to be the main reason for the failure of Carnap's project of building a logic of induction: its connection with the problem of justification. We then considered from a conceptual point of view the possibility of building a purely descriptive logic of induction which would avoid Carnap's flaws. An attempt to implement the suggestions shown in Section 7 can be found in Silvestre (2005).

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[41] An instance of such axioms would be what we could call inductive implication transitivity axiom: if α inductively implies β and β inductively implies φ, then α inductively implies φ.

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