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Application of Directional Statistics in Cartographic Quality Control

Aplicação da Estatística Direcional no Controle de Qualidade Cartográfico

Victor Alvarenga Carvalho[1]; Afonso de Paula dos Santos[2]; Marconi Martins Cunha[3]; Lígia da Silva Barbosa[4]; William Rodrigo Dal Poz[5]; Nilcilene das Graças Medeiros[6]; Julio Cesar de Oliveira[7]

[1] Federal University of Viçosa, Department of Civil Engineering, Viçosa/MG, Brazil, Email: victor.alvarenga@ufv.br
ORCID: https://orcid.org/0009-0009-0920-0707

[2] Federal University of Viçosa, Department of Civil Engineering, Viçosa/MG, Brazil, Email: afonso.santos@ufv.br
ORCID: https://orcid.org/0000-0001-7248-4524

[3] Federal University of Viçosa, Department of Civil Engineering, Viçosa/MG, Brazil, Email: marconi.cunha@ufv.br
ORCID: https://orcid.org/0000-0003-0797-8530

[4] Federal University of Viçosa, Department of Civil Engineering, Viçosa/MG, Brazil, Email: ligia.barbosa@ufv.br
ORCID: https://orcid.org/0000-0003-1929-8110

[5] Federal University of Viçosa, Department of Civil Engineering, Viçosa/MG, Brazil, Email: william.dalpoz@ufv.br
ORCID: https://orcid.org/0000-0001-9532-3643

[6] Federal University of Viçosa, Department of Civil Engineering, Viçosa/MG, Brazil, Email: nilcilene.medeiros@ufv.br
ORCID: https://orcid.org/0000-0003-0839-3729

[7] Federal University of Viçosa, Department of Civil Engineering, Viçosa/MG, Brazil, Email: oliveirajc@ufv.br
ORCID: https://orcid.org/0000-0003-0894-5597

Abstract: A cartographic product is considered positionally accurate when it is precise and not biased. Trend analysis is generally performed using the Student's t test, which requires that the sample follows a normal distribution. One solution is the use of circular descriptive statistics based on Directional Mean and Circular Variance, which do not assume normality. However, analyzes of circular descriptive statistics may be inadequate and due care is not taken. Therefore, it is necessary to add more robust processes on directional statistics for positional assessment, such as joint analyzes of the main descriptive parameters, application of statistical tests of normality and directional uniformity. The use of these processes is evaluated in this work with the aim of identifying solutions to avoid failures in trend detection in cartographic data. These failures were exemplified by simulated models, which demonstrated situations in which the analysis of Directional Mean and Circular Variance may not be effective. It was also proposed to use statistical tests, joint analysis of several descriptive parameters and graphs of directional statistics as a complement to detect trends. This procedure was applied to two Digital Surface Models (DSM), with trends detected in both.

Keywords: Directional Statistics; Positional Accuracy; Trend Detection.

Resumo: Um produto cartográfico é considerado acurado posicionalmente quando é preciso e não tendencioso. A análise de tendência geralmente é realizada utilizando-se o teste t de Student, que tem como exigência que a amostra siga distribuição normal. Uma solução é o uso de estatísticas descritivas circulares a partir da Média Direcional e Variância Circular, que não pressupõem a normalidade. Entretanto, as análises sobre as estatísticas descritivas circulares podem ser inadequadas não sendo adotados os devidos cuidados. Por isso, é preciso acrescentar processos mais robustos sobre a estatística direcional para a avaliação posicional, como análises em conjunto dos principais parâmetros descritivos, aplicação de testes estatísticos de normalidade e uniformidade direcional. A utilização desses processos é avaliada nesse trabalho com o objetivo de identificar soluções para evitar falhas na detecção de tendência em dados cartográficos. Essas falhas foram exemplificadas por modelos simulados, que demonstraram situações em que a análise da Média Direcional e Variância Circular podem não ser efetivas. Também foi proposto a utilização de testes estatísticos, análise conjunta de diversos parâmetros descritivos e gráficos da estatística direcional como complemento para a detecção de tendências. Este procedimento foi aplicado a dois Modelos Digitais de Superfícies (MDS), havendo a detecção de tendências em ambos.

Palavras-chave: Estatística direcional. Acurácia Posicional. Detecção de Tendência.

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1. Introduction

The use of a cartographic product is related to the various needs of the user and, regardless of its application, the cartographic data set must meet the minimum quality prerequisites. When there is a need to extract geometric measurements from a map base, special attention must be paid to checking that the information has spatial quality.

Verifying the spatial accuracy of a cartographic product must comply with standards and norms that use statistical methods to assess positional discrepancies between the product being assessed and a reference product (IBGE, 2019). In Brazil, Decree No. 89.817 (BRASIL, 1984) is used as a basis, which established parameters for classifying a product in terms of its planimetric and altimetric components, through the Standard Cartographic Accuracy (PEC) and Standard Error (EP) tolerances, which assess the magnitude of the discrepancies. It is also worth noting more recent standards, such as the Technical Specification for Structuring Vector Geospatial Data (ET-EDGV) (CONCAR, 2011) and the Technical Specification for Quality Control of Geospatial Data (ET-CQDG) (DCT/DSG, 2016), which apply the concept to digital map bases, adopting the term PEC-PCD (Cartographic Accuracy Standard-Digital Cartographic Standard) instead of PEC and keeping the term EP.

In addition to the magnitude of the discrepancies, it is of fundamental importance to check for the presence of systematic effects by evaluating the trend, which is a component of positional accuracy, often checked using the Student's t hypothesis test (SANTOS et al., 2016). The parameters used in this assessment are based on the assumption that the sample has a Normal distribution, which does not necessarily occur in cartographic data (CUARTERO et al., 2010).

Given these limitations, as an alternative to detecting trends, analyses can be carried out using directional statistics, in which the vector of discrepancies is used, defined by its magnitude and direction (CUARTERO et al., 2014; POLO; FELICÍSIMO, 2010; SANTOS et al., 2016; VITTI et al., 2018; PAULA and CARVALHO, 2024). This statistic allows the identification of behaviors not explained by linear statistics, such as the possibility of making inferences related to the analysis of directions and the behavior of a phenomenon on spherical or circular surfaces, as occurs in some applications in biology, geology and meteorology (FISHER, 1985; JAMMALAMADAKA; SEAGUPTA, 2001). In this scenario, it is common to address only the directions of the vectors, with the focus on detecting the characteristics of the sample distribution over a circumference or unit sphere (MARDIA; JUPP, 2000).

Directional statistics offers possible solutions and applications for analyzing positional accuracy. Thus, we can envision the application of circular statistics for trend detection, as well as the possibility of integrating planimetry and altimetry by analyzing the distribution of the vector of discrepancies on the sphere.

However, analysis of circular descriptive statistics can be inadequate if care is not taken, as in the case of altimetric evaluation or bimodal data. In these cases, it is necessary to add more robust processes to the directional statistics. This study therefore evaluated the processes involved in this concept, such as: joint analysis of the main descriptive parameters, application of statistical tests for normality and directional uniformity, and graphical analysis of the behavior of vectors, with the aim of identifying solutions to avoid failures in detecting trends in cartographic data.

2. Methodology

To process the data and apply the methodologies evaluated in this work, the following materials were used: R software (R CORE TEAM, 2024), packages: VecStatGraphs3D (FELICISIMO et al., 2015) and CircStats (AGOSTINELLI; LUND, 2018), GeoPEC software version 3.5.2 (SANTOS, 2019) and QGIS software version 3.10.11 (QGIS, 2020).

To carry out the experiments, we used the three-dimensional discrepancy data from Santos (2015) (Figure 1), which is data free of outliers, which were removed using the Boxplot. The discrepancies originate from the analysis of 241 points from the SRTM DSM (X band) and 248 points from the Aster GDEM DSM (version 2), whose reference was an DSM obtained by digital aerophotogrammetry. Obtaining three-dimensional discrepancies in the DSMs was made possible by applying the technique of extracting homologous points from ridge lines and numerical hydrography lines (SANTOS et al., 2020). For more details on DSM and DTM (Digital Terrain Model), it is worth highlighting some works, such as those by Pessi et al. (2021) and Caldeira et al. (2023).

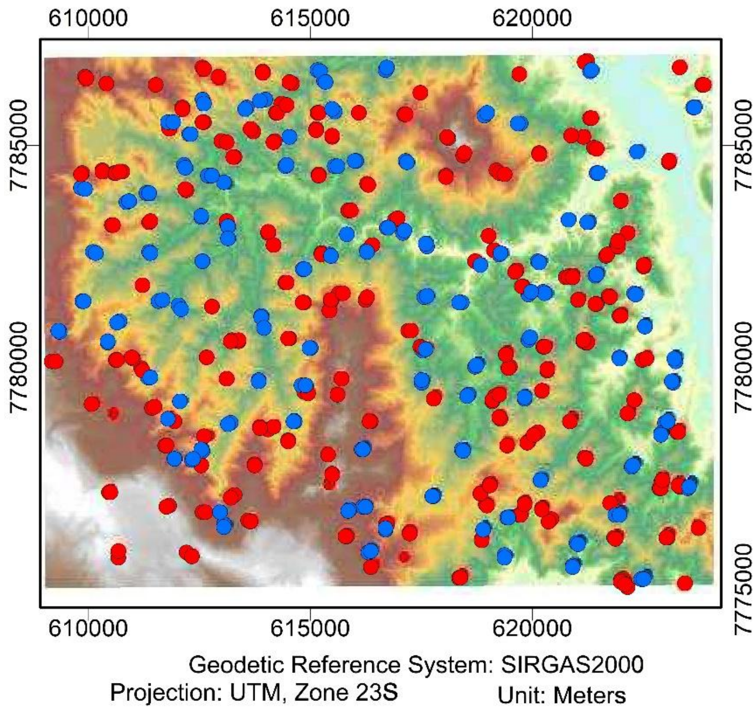

Figure 1 – 3D checkpoints in the study area. Red circles correspond to ridge points and blue circles to hydrography confluence points.
Source: Adapted from Santos et al. (2015).

The methodological process adopted for trend analysis can be subdivided into two phases, as exemplified in Figure 2: the traditional one, used in the GeoPEC software (SANTOS, 2019), and the alternative one, based on the directional statistics proposed in this work.

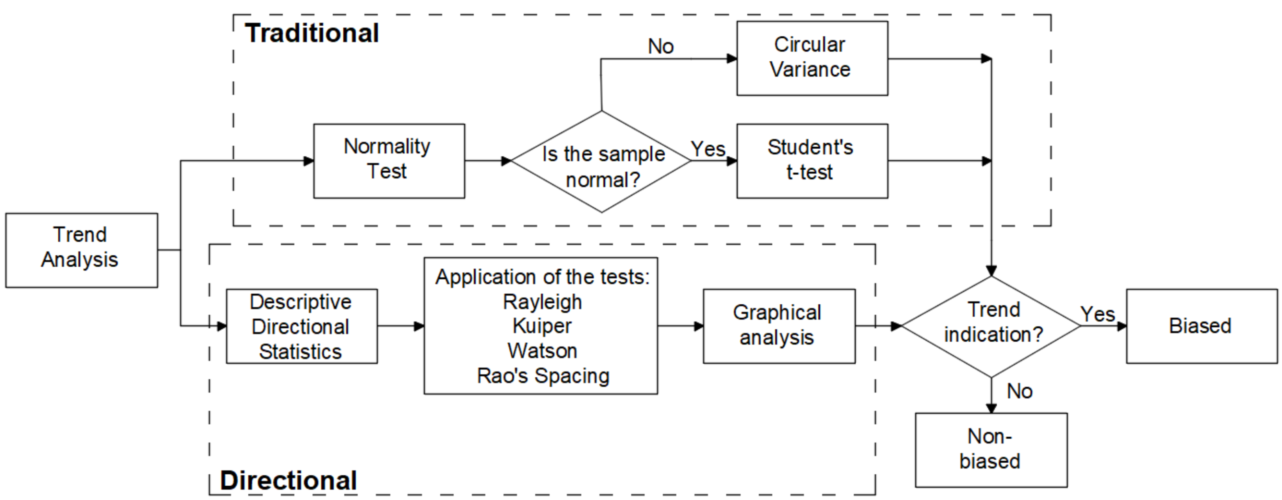

Figure 2 – Methodological flowchart for trend analysis in cartographic products.
Source: Authors (2024).

2.1. Traditional Analysis

To apply the traditional methodology, we used positional discrepancy data from the SRTM-X and GDEM DSM (SANTOS et al., 2015) and the GeoPEC software (SANTOS, 2019). As the sample set had been previously removed of outliers, the planimetric and altimetric normality tests could be carried out. To this end, the Shapiro-Wilk test was used with a 90% confidence level.

The next step was to detect trends. To do this, we used the Student's t-test with a 90% confidence level. This test gives the tabulated Student's t value and the calculated value for comparison: if the calculated Student's t module is less than the tabulated Student's t, it is concluded that the average of the positional discrepancies is statistically equal to zero. Therefore, the product being evaluated has no trend.

The second part of the traditional trend assessment uses directional statistics, applicable only to the planimetric component. The process is carried out by evaluating two variables: the Directional Mean, which describes the trend direction of the positional discrepancy vectors, and the Circular Variance, which measures the variability of these directions. At this stage, no trend is considered to be present when the Circular Variance value is greater than 0.5, indicating that the positional discrepancy vectors are distributed in such a way as to suggest the absence of a predominant direction.

In addition to the trend analysis, the accuracy analysis of the cartographic products was also carried out, based on the parameters of Decree 89.817 (BRASIL, 1984) and ET-ADGV (Technical Specification for the Acquisition of Geospatial Vector Data) (CONCAR, 2011 and DCT/DSG, 2016). This analysis consists of checking that the RMS (Root Mean Square) of the sample of discrepancies is less than or equal to the Standard Error (SE), and that 90% of the positional discrepancies are less than or equal to the PEC and/or PEC-PCD (the latter in the more modern terminology of CONCAR, 2011 and DCT/DSG, 2016).

Finally, a map was generated to spatialize the vectors of planimetric discrepancies. To do this, a file was produced containing the position of the sample, the value and the orientation of the positional discrepancies.

2.2. Directional Analysis

In the directional analysis, the calculations were made from the unit vector of discrepancies, which are stored in a file, then used as input in the R software (R CORE TEAM, 2024). The VecStatGraphs3D and CircStat function packages were used for the statistical calculations.

The descriptive parameters of the directional statistics: Resulting mean length ($\bar{R}$), Mean direction ($\bar{\theta}$) calculated in relation to the N axis, Circular variance (V), Circular standard deviation (v), Concentration parameter (κ), Asymmetry ($\bar{s}$) and Kurtosis ($\bar{k}$) were calculated and analyzed together. The Rayleigh, Kuiper, Watson (Uniform and Von Mises) and Rao's tests were also applied. After the statistical analyses, graphs were generated to enable visual analysis of the behavior of the vectors together with the statistical results.

Before carrying out the practical application, the behavior of circular statistics was checked by three simulated models, consisting of 10 points spread over the surface of a sphere in different ways: (i) in the first model the circumference was divided into equal intervals, thus subdividing the sample into 10 identical arcs; (ii) the second model distributes the 10 points on the circumference in an interval of 10° centered and symmetrical to the origin, i.e. five samples to the right of the origin and five samples to the left and; (iii) the third model represents a bimodal sample with five points distributed in an interval of 5° and the other five points rotated by 180° in relation to the first. The representations of these models are shown in Figure 3.

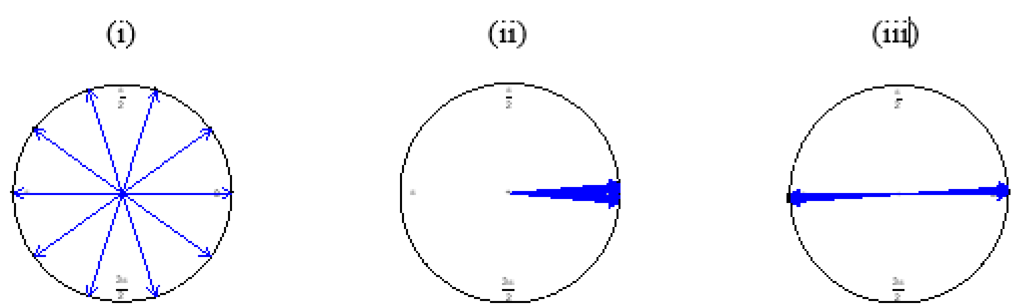

Figure 3 – Vector representation of the simulated models: (i) uniform; (ii) concentrated; (iii) bimodal.
Source: Authors (2024).

The characteristics of the simulated models are intended to describe the evaluation power of the proposed methodology in extreme cases and in detecting possible grouping or dispersion of discrepancies. It is expected that the first model will be identified as a uniform distribution; the second model, as a non-uniform distribution, but with the characteristics of a normal distribution, with symmetry in relation to the mean. The third model aims to evaluate the statistical test when a sample is bimodal.

For the practical application of evaluating the SRTM and GDEM DSMs, the same tests and parameters were used as for the simulated model, when calculating the descriptive variables and statistical tests. However, it was necessary to treat the three-dimensional, non-unitary discrepancy data. To do this, the unit component of the discrepancies was calculated according to Eq. (1), (2) and (3).

$$x_{ij} = \frac{Dx_{ij}}{d_{3D}} \tag{1}$$

$$y_{ij} = \frac{Dy_{ij}}{d_{3D}} \tag{2}$$

$$z_{ij} = \frac{Dz_{ij}}{d_{3D}} \tag{3}$$

Where:

- $Dx_{ij}, Dy_{ij}, Dz_{ij}$ are the components of the positional discrepancies;
- x_{ij}, y_{ij}, z_{ij} are the unit components of the positional discrepancies;
- d_{3D} is the three-dimensional discrepancy.

It should be noted that for two-dimensional analysis, the 3D data must be broken down into two components. To do this, the coordinates in the Cartesian system must be transformed into the spherical polar coordinate system using Eqs. (4) and (5). This process results in two circumferences: (i) azimuthal, with the calculation of the colatitude (θ), which will be responsible for representing the X and Y plane, and (ii) zenithal, from the latitude (φ), which parameterizes the vertical discrepancies.

$$\theta = \tan^{-1}\left(\frac{y_{ij}}{x_{ij}}\right) \tag{4}$$

$$\varphi = \tan^{-1}\left(\frac{z_{ij}}{\sqrt{x_{ij}^2 + y_{ij}^2}}\right) \tag{5}$$

It is also worth noting the axial nature of the latitude, since the denominator will always be positive, restricting the angle to the range 0 to 180 degrees, which is coherent since it will only evaluate the variation between the note-south axis of the sphere. Given this characteristic, statistical tests will not be applied to the latitude, only the graphical analysis and the descriptive parameters of the directional statistics.

3. Results and discussion

3.1. Traditional Analysis

The set of positional discrepancies was checked for normality using the Shapiro-Wilk test at a 90% confidence level. The results showed that in altimetry there was no normality in both products and, for the planimetric components, only the East component (X) of the SRTM-X did not show normality (Table 1).

Table 1 – Results of the normality test for the DSM, according to the Shapiro-Wilk test.

Component		**SRTM-X**		**GDEM2**	
Planimetric	X	p-value = 0.0143	Not normal	p-value = 0.5827	Normal
	Y	p-value = 0.2818	Normal	p-value = 0.37432	Normal
Altimetric		p-value = 0.053	Not normal	p-value = 0.006	Not normal

Source: Authors (2024).

Therefore, as they do not follow a normal distribution, the altimetry and east component of the SRTM-X do not meet the minimum requirements for analysis using the Student's t-test. Therefore, Student's t-test was applied to check for trends only for the other components (Table 2). As an alternative to detecting trends in the non-normal case, the spatial statistics in the GeoPEC software were applied. These statistics did not indicate the presence of trends, since the Circular Variance was greater than 0.5.

Table 2 – Parameters and results of the planimetric and altimetric trend tests.

Trend	**SRTM-X**		**GDEM2**	
Planimetric (Student's t)	t_{tab} = 1.65 $t_{calc\ N}$ = 5.13	Biased	t_{tab} = 1.65 $t_{calc\ E}$ = 3.65 $t_{calc\ N}$ = 6.71	Biased
Planimetric (spatial statistics)	$\bar{\theta}$ = 54.31° V = 0.591	Not Biased	$\bar{\theta}$ = 32.57° V = 0.66	Not Biased
Altimetric (Student's t)	t_{tab} = 1.65	Not applicable	t_{tab} = 1.65	Not applicable

Source: Authors (2024).

For planimetry, the spatial statistics showed no trend. However, Student's t-test indicated the presence of systematic effects in the set of discrepancies. Therefore, considering the most rigorous result, the spatial data was considered biased. The result for altimetry was inconclusive. This was because Student's t-test was not applied, since the sample did not follow a normal distribution, and spatial statistics are not applicable to discrepancies arising from altimetry data.

Following the parameters of Decree 89.817/ET-ADGV (BRASIL, 1984, CONCAR, 2011 and DCT/DSG, 2016), both products were classified in class D on a scale of 1:100,000. For altimetry, considering the vertical equidistance between contour lines of 50 m, the classification in class B was obtained, as can be seen in Table 3.

Table 3 – DSM evaluation results for planimetric and altimetric accuracy, according to the recommendations of Decree 89.817 (BRASIL, 1984)/ET-ADGV and ET-CQDG (CONCAR, 2011 and DCT/DSG, 2016).

Product	**Component**	**Scale (Class)**	**PEC(m)**	**% (di ≤ PEC)**	**EP(m)**	**RMS (di)**
SRTM-X	Altimetric	1:100.000 (B)	25	99%	16.7	12.04
	Planimetric	1:100.000 (D)	100	92%	60	58.31
GDEM2	Altimetric	1:100.000 (B)	25	100%	16.7	9.60
	Planimetric	1:100.000 (D)	100	92%	60	59.43

Source: Authors (2024).

The results led to the conclusion that the product is not accurate, since there is a trend. The acceptance of both products in class A is conditional on the use of scales smaller than 1/500,000, where the RMS is smaller than the EP and 100% of the discrepancies are smaller than the PEC.

For the visual analysis, the discrepancy map was created, which spatially represented the displacement between the evaluated and reference products. It was possible to see that there are regions with greater discrepancies. However, as this was a large sample, the large number of vectors meant that the visual analysis was not trivial in terms of detecting a preferential direction for the discrepancies.

3.2. Directional Analysis

The directional (circular) analysis was first applied to the simulated models. In the first sample set, all the descriptive statistics indicated uniformity of the vectors, confirming the expected value. For the second distribution, the sample was found to be concentrated in one direction with a resulting mean length close to 1. Asymmetry, with a value of 0, indicated symmetry around the mean, which is one of the characteristics of the Von Mises distribution. Thus, this simulation showed the possibility of a data set being biased and following the circular normal or Von Mises distribution.

In the third simulation, which simulated bimodal data, the length of the directional mean, the circular variance and the concentration parameter did not follow the expected pattern. Due to this characteristic, it is not possible to generalize the observation that when the sample variance is close to 0, the distribution is uniform as suggested by Santos et al (2016). As for kurtosis, among the descriptive statistics in this simulation, it was the only one that indicated that the data was not uniform. Therefore, this reinforces the assertion that analyses should be carried out considering all the parameters suggested in this research simultaneously. These results are shown in Table 4.

Table 4 – Parameters of the descriptive circular statistics of the proposed simulated models.

Simulated Model	**(i)**	**(ii)**	**(iii)**
Resulting average length ($\bar{R}$)	0	0.99	0
Medium direction ($\bar{\theta}$)	0	360°	0
Circular variance (V)	1	0.001	1
Circular standard deviation (v)	8,571	0.057	8.52
Concentration parameter (κ)	0	298.822	0
Asymmetry ($\hat{s}$)	0	0	-0.069
Kurtosis ($\hat{k}$)	0	-2.757	-0.996

Source: Authors (2024).

With regard to the statistical tests, it was observed that in both the first and second simulations, all the results converged on the same indication regarding the acceptance or rejection of the initial hypothesis. However, in the third group of simulated data, there was no agreement between the statistical tests (Table 5). This is justifiable, since the result of a test is linked to the adequacy and power of each test in detecting the behavior of the sample.

In bimodal samples, it is not advisable to use the Rayleigh test (JAMMALAMADAKA; SEAGUPTA, 2001). Therefore, it is justifiable to accept the hypothesis of uniformity when rejection was expected (type I error). The Rao's Spacing test, which is based on the length of arcs, is less susceptible to type I error in bimodal samples, and showed a result of non-uniformity (Table 5).

Table 5 – P-value of the statistical tests of uniformity and Von Mises distribution for the simulated models and analysis of the results as to whether or not they follow the distributions tested.

Conceptual model	**(i)**	**(ii)**	**(iii)**
Rayleigh	= 1	= 0	= 1
Kuiper	> 0.15	< 0.01	< 0.10
Watson (Uniform)	> 0.10	< 0.01	< 0.05
Watson (Vom Mises)	> 0.10	> 0.10	< 0.01
Rao’s spacing	> 0.10	< 0.001	< 0.001
Uniformity	Yes	No	No
Von Mises	Yes	Yes	No

Source: Authors (2024).

The DSM evaluation was subdivided into three: azimuth, zenith and 3D, with the aim of obtaining a complete analysis and checking the feasibility of each of these analyses (Table 6 and Table 7).

Table 6 – Parameters of the descriptive circular statistics of the azimuths of the DSM discrepancies.

DSM	**GDEM2**	**SRTM-X**
Resulting average length ($\bar{R}$)	0.330	0.408
Medium direction ($\bar{\theta}$)	32.48º	54.31°
Circular variance (V)	0.669	0.591
Circular standard deviation (v)	1.487	1.337
Concentration parameter (κ)	0.807	1.017
Asymmetry ($\hat{s}$)	0.248	8.341
Kurtosis ($\hat{k}$)	0.140	137.598

Source: Authors (2024).

Table 7 – P-value of the statistical tests of uniformity and Von Mises distribution of the azimuths of the DSM discrepancies.

DSM	**GDEM2**	**SRTM-X**
Rayleigh	= 0	= 0
Kuiper	< 0.01	< 0.01
Watson (Uniform)	< 0.01	< 0.01
Watson (Vom Mises)	< 0.01	< 0.05
Rao’s spacing	< 0.01	< 0.001
Uniformity	No	No
Vom Mises	No	No

Source: Authors (2024).

When analyzing the descriptive statistics of the azimuths, it can be seen that in the GDEM2 product the discrepancies do not follow a uniform distribution, as it has a significant circular standard deviation and a high concentration parameter. As for SRTM-X, the kurtosis obtained a high value, indicating that the sample is not uniformly distributed. Another noteworthy parameter is asymmetry, which indicates that the sample tends not to be normal (Table 6). These results were confirmed statistically as shown in Table 7.

In addition to the statistical analysis, the rose diagrams showed (Figure 4) that the sample of discrepancies has a preferred direction, since it can be seen that the frequency is not uniform throughout the sphere. The positional discrepancy graphs show how the vectors behave, and make it possible to visualize the behaviour of the resulting vector, verifying the magnitude and direction of the concentration of discrepancies in one direction.

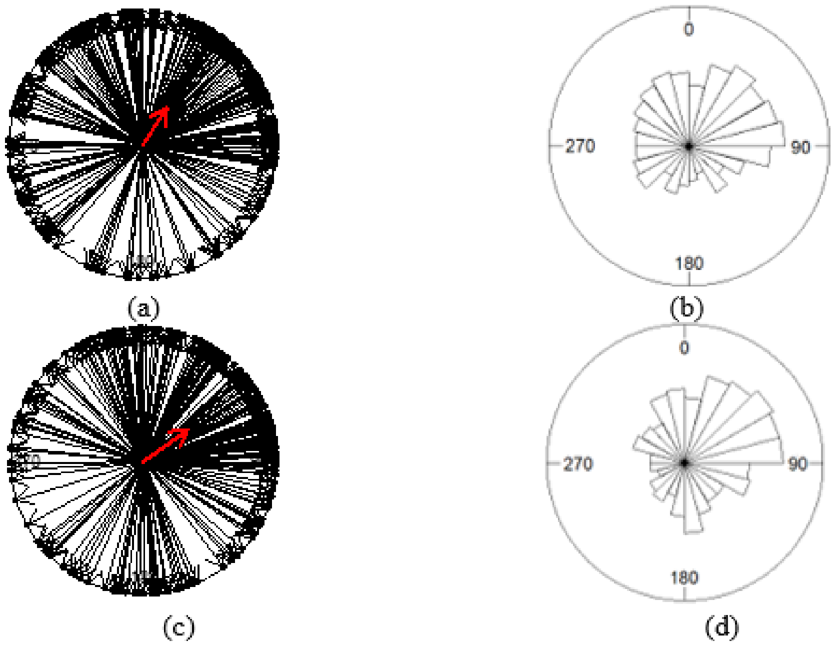

Figure 4 – Representation of the planimetric component of the unit positional discrepancies of the DSM, by means of a vector in black arrows and a resultant in red, and a rose diagram relative to the frequency partitioned into 30 intervals: (a) GDEM2 vector; (b) GDEM2 rose diagram; (c) SRTM-X vector; (d) SRTM-X rose diagram.
Source: Authors (2024).

The zenith angle was analyzed using the descriptive parameters in Table 8. Among the descriptive parameters, the length of the high directional mean ($\bar{\theta}$) stands out. The parameters confirmed that the sample of discrepancies is concentrated, with kurtosis not close to 0 and a high concentration parameter.

Table 8 – Parameters of the descriptive circular statistics of the zenith angles of the DSM discrepancies.

DSM	**GDEM2**	**SRTM-X**
Resulting average length ($\bar{R}$)	0.884	0.882
Medium direction ($\bar{\theta}$)	82.93º	105.91°
Circular variance (V)	0.116	0.032
Circular standard deviation (v)	0.496	0.501
Concentration parameter (κ)	4.993	4.828
Asymmetry ($\hat{s}$)	-1.140	2.856
Kurtosis ($\hat{k}$)	5.073	4.430

Source: Authors (2024).

For the 3D analysis, the main statistics were calculated, which indicated that there was no evidence that the sample of discrepancies was concentrated. In addition, the Rayleigh statistical test concluded moderate evidence against the uniformity of the data (Table 9). Therefore, the visual analysis of the vectors (Figure 5) was essential in this case, where it was possible to see that the sample is unimodal. However, it was not possible to infer the presence of a single three-dimensional direction that showed a trend.

Table 9 –Parameters of the descriptive spherical statistics of the discrepancies relative to the DSM and the P-value of the statistical test of uniformity.

	GDEM2	SRTM-X
Length of directional average ($\bar{R}$)	0.352	0.465
Colatitude ($\bar{\theta}$)	69.59º	124.01°
Longitude ($\bar{\varphi}$)	57.26º	34.15°
Concentration parameter (κ)	1.536	1.861
Rayleigh uniformity test (P-value)	= 0.05	= 0.05

Source: Authors (2024).

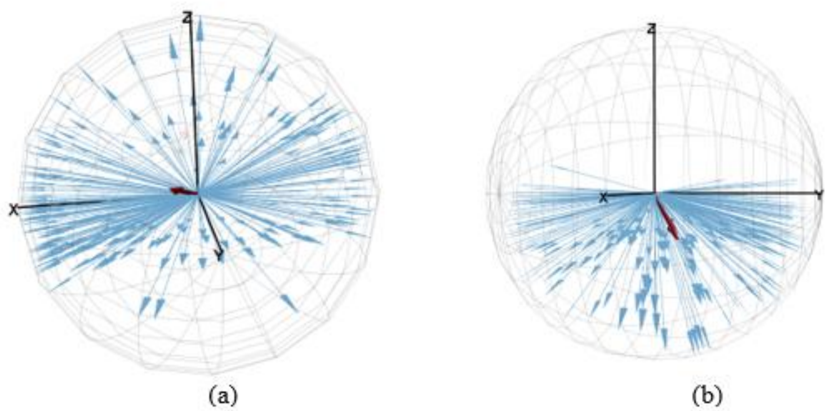

Figure 5 – Unit vectors representing the directions of positional discrepancies symbolized by blue arrows and the vector resulting from the direction of the discrepancies in red: (a) Relative to the GDEM2 DSM and (b) the SRTM-X. Source: Authors (2024).

The results detected the presence of a trend in cartographic data using directional statistics, through a joint analysis of graphs, descriptive variables and statistical tests. It should be noted that when applied in isolation, the tests or descriptive variables may present inadequate results and not adequately represent the presence of a trend.

4. Final considerations

In view of the results, it can be concluded that the use of circular statistics to analyze cartographic data tends to contribute to the results of trend analysis, since it is possible to infer the distribution of positional discrepancy vectors. Circular statistics not only complements traditional analyses, but also provides an alternative in cases where map data does not follow the Normal distribution, where it is not possible to apply Student's t-test. This is a common situation with spatial data. Furthermore, considering that there are now computers with greater effort and processing capacity, we suggest using the normal function, i.e. the Normal N-test, in future work instead of the Student's t-test.

The feasibility of applying the statistics and the appropriate precautions to be taken were verified through the simulated models. These results showed that the descriptive circular statistics should be used as an indication of the behavior of the vectors on the circumference. They also showed that statistical tests should be used as a way of confirming the evidence gathered about the sample. Visual analysis using graphs proved to be an important tool for confirming the numerical results obtained in the uniformity check.

Another important point identified in the analysis of the simulated models was the behavior of the statistical tests in bimodal cases. In these cases, it was found that the Rayleigh test has low power, and also warns of the care to be taken when carrying out analyses based on variance.

The application of the circular methodology to the positional discrepancies of the GDEM2 and SRTM-X DSM made it possible to verify the presence of a trend in the data, since the analyses of the planimetric and altimetric components detected a concentration of discrepancies in a certain direction. Therefore, the proposal proved to be a viable alternative for concluding that both products are biased, both in altimetry and planimetry, with emphasis on altimetry, where the trend was more evident, due to the greater concentration of discrepancies.

In general, the detection of a global trend in a product was verified using circular statistics. However, the evaluation of a cartographic product does not only involve trend detection, but also a joint analysis between trend and accuracy. This highlights the need for future work to develop joint analysis of the modulus and direction of discrepancies applied to circular statistics, as well as further studies into three-dimensional analysis, with the aim of implementing more robust methodologies for trend detection.

Finally, considering the increasing popularization of geotechnologies, there are more simplified analyses and possibilities, taking into account that many data are obtained with good precision and a concentration of errors that make up a leptokurtic distribution. This applies fundamentally to data obtained by GNSS (Global Navigation Satellite System) and also to data obtained by RPAs (Remotely Piloted Aircraft System), which would certainly not pass the normality test, which suggests looking for new research alternatives and solving the problem.

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