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A fuzzy logic approach for diagnosing and planning the leveling network of the Brazilian Geodetic System

Uma abordagem baseada em lógica fuzzy para o diagnóstico e planejamento da rede de nivelamento do Sistema Geodésico Brasileiro

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Abstract: The leveling network consists of benchmarks that represent the vertical component of a geodetic system. This network is essential for the planning and execution of engineering projects. Its maintenance and densification require detailed planning to ensure the efficient allocation of resources. In this sense, to optimize the diagnosis and planning of geodetic campaigns, assisting decision-making in the face of budgetary and operational limitations, a methodology based on fuzzy logic was proposed, which deals with uncertainties and subjectivities in the decision-making process. Thus, in the development of this research, fuzzy logic was applied using the Mamdani method, involving fuzzification, inference, and defuzzification, resulting in the hierarchical prioritization of field activities into high, medium, and low categories. Notably, the results identified 9,155 destroyed benchmarks and 26,322 benchmarks classified as not found, amounting to 49.6% of the leveling network. This situation could be more critical, as 19,388 of the 22,524 benchmarks marked as 'good' in the geodetic database and classified as high-priority for physical verification have not been visited in over 20 years. The approach provided a systemic vision to support planning and resource optimization, strengthening strategic decision-making.

Keywords: Fuzzy logic; Leveling network; Diagnosis.

Resumo: A rede de nivelamento é um conjunto de marcos geodésicos denominados referências de nível (RRNN), que materializam a componente vertical de um sistema geodésico, sendo essencial para o planejamento e execução de obras de engenharia. Sua manutenção e densificação demandam um planejamento detalhado para empenho adequado de recursos. Nesse sentido, para otimizar o diagnóstico e o planejamento de campanhas geodésicas, auxiliando a tomada de decisão diante de limitações orçamentárias e operacionais, foi proposta uma metodologia baseada em lógica fuzzy, que lida com incertezas e subjetividades do processo decisório. Assim, no desenvolvimento da corrente pesquisa foi aplicada a lógica fuzzy pelo método Mamdani, envolvendo a fuzzificação, inferência e defuzzificação, tendo como resultado a priorização hierárquica de atividades de campo em alta, média e baixa. Como destaque dos resultados, identificou-se que RRNN destruídas somam 9.155 e as não encontradas, 26.322, o que corresponde a 49,6% da rede de nivelamento. Esse diagnóstico pode ser pior, já que das 22.524 RRNN com o status bom no banco de dados geodésicos e classificadas como prioridade alta de verificação da realidade física, 19.388 não são visitadas há mais de 20 anos. A abordagem proporcionou uma visão sistêmica de subsídio ao planejamento e à otimização de recursos, fortalecendo a tomada de decisão estratégica.

Palavras-chave: Lógica fuzzy; Rede de nivelamento; Diagnóstico.

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1. Introduction

The Brazilian height network, known as the High Precision Altimetric Network (Rede Altimétrica de Alta Precisão – RAAP), is the vertical component of the Brazilian Geodetic System (BGS), responsible for transporting heights throughout the national territory. The RAAP has been developed along the road network since 1945 (ALENCAR, 1968) and is currently represented by 71,475 benchmarks (BMs), which provide a vertical reference infrastructure for a wide range of activities where knowledge of water flow is crucial.

During its development, due to the impossibility of transporting heights using geometric leveling techniques to the state of Amapá, as there are no available land routes through the extremely wide mouth of the Amazon River, the network, previously connected only to the Imbituba Datum, began to rely exclusively on the Santana Datum to serve that state in 1980 (LUZ and GUIMARÃES, 2001).

Over the past decade, the main developments related to the densification of the RAAP can be organized into three moments. Between 2014 and 2016, the network was revitalized in the state of Amapá (PORTAL DO ESTADO DO AMAPÁ, 2016). Between 2017 and 2019, a coastal reference network was implemented on the coast of the state of Rio de Janeiro, focused on integrating altimetry and bathymetry research (SOARES; SANTOS; LUZ, 2019). And between 2018 and 2022, the height connection of the Marabá continuous GNSS station stood out, the calculations for which were recently released (IBGE, 2024).

Like any leveling network, the RAAP underwent several recalculations due to the incorporation of observations, correction of inconsistencies, and new observation and calculation techniques. However, the most recent readjustment, which took place in 2018, profoundly modernized it. The increased availability of gravimetric observations throughout the country enabled readjustment with geopotential numbers, following the recommendations of the Geodetic Reference System for the Americas (SIRGAS, 2021), in agreement with the International Association of Geodesy (IAG), toward the adoption of the future International Height Reference System (IHRS) and its corresponding International Height Reference Frame (IHRF) (IBGE, 2019). It is also noteworthy that, as of this readjustment, heights categorized as "normal-orthometric" become normal heights, using the hgeoHNOR2020 height conversion model (IBGE, 2023).

The maintenance and expansion of the RAAP involve complex activities that require significant human and financial resources and a low production scale. Unfortunately, techniques successful in other countries, such as motorized leveling (POETZSCHKE, 1980; VESTØL et al., 2014; IGN, 2019), have not been used to expand the RAAP to date.

Looking at the global scenario, it is clear that the demand for the vertical component remains high and essential in crustal movement research, geoid model verification, and infrastructure implementation, such as ports, highways, and water and sewage networks, among others. Examples include the research by Chao, Overton, and Nelson (2006) on the control of expansive soils; the implementation of a 360-km line in Colorado (USA) in 2017 to qualify the accuracy of various geoid models (VAN WESTRUM et al., 2021; WANG et al., 2021); and the project to implement a leveling network in France, conceived in 2008, with an estimated duration of 12 years, covering the country in latitudinal and longitudinal lines. This initiative, in addition to connecting tide gauges, represents a supporting skeleton for the French height network (IGN, 2019). In contrast, there is the Canadian case, which abandoned its leveling network and replaced it exclusively with a gravimetric model (SANTOS, 2015).

Returning to Brazil, the RAAP, like any network materialized on the ground, constantly suffers from plate thefts combined with unpunished vandalism. Despite the "protected by law" label clearly appearing on the paper and on the plate, its effectiveness in maintaining the network is practically zero. In many cases, urban expansion and highway widening, which the RAAP serves, contribute to its demise. Therefore, considering the large volume of data, an adequate diagnosis capable of contributing to prioritizing field activities can benefit from the use of artificial intelligence (AI), supporting more assertive decision-making by managers and thus enabling the appropriate use of resources for maintenance and densification of the RAAP.

In this context, it is noteworthy that the application of AI has been expanding in geodesy. Studies in gravimetry show that machine learning and deep learning algorithms contribute to the fusion of heterogeneous data, noise reduction, and modeling of temporal variations in the gravity field, increasing the accuracy of geoid models (IDOWU and ILESANMI, 2025). At the same time, artificial neural networks have been applied to reference system transformations, with performance comparable or superior to classical methods (HUSSEIN; ALHAMADANI; HUSSEIN, 2025). Furthermore, international initiatives to modernize geodetic networks demonstrate the potential of using automated, AI-based methodologies for quality control and adjustment of large-scale GNSS observations (LEE and YUN, 2025).

Complementarily, fuzzy inference techniques have shown promise in various geodetic and environmental applications. Permana et al. (2024) used a fuzzy system to validate sea level anomalies by integrating satellite altimetry and tide gauges, increasing the reliability of estimates in vulnerable coastal regions. Similarly, recent research indicates that fuzzy algorithms can improve the accuracy of GPS receivers in different scenarios, reducing errors and validating coordinates obtained by low-cost sensors (HARIYONO; MARWANTO; ALIFAH, 2024).

This article presents a methodological proposal based on fuzzy logic for the diagnosis and planning of RAAP field activities. The approach seeks to provide the technical team with a systemic tool that complements the criteria currently based on expert experience, reducing subjectivity in the decision-making process. However, a lack of specific resources and a view dependent on expert experience can compromise the effectiveness of these actions, especially given the growing demand for high-precision geodetic data in sectors such as infrastructure, security, and the environment.

2. Theoretical Foundation

Prioritizing geodetic campaigns is essential to ensure resource optimization and efficiency in field operations, where technical and strategic criteria are rigorously analyzed. In this work, fuzzy logic was adopted because it offers an approach that allows for the subjectivity and uncertainty inherent in the decision-making process.

Fuzzy logic, as presented by Zadeh (1965), is an extension of traditional Boolean logic that deals with uncertainty and imprecise information, allowing for representations closer to human reasoning. In other words, unlike traditional logic, fuzzy logic uses the idea that all things admit degrees of relevance. Thus, it attempts to model the sense of words, decision-making, or common sense of human beings (MARRO et al., 2010).

Fuzzification through membership functions defines the degree of membership of an element in a fuzzy set, allowing for the mathematical representation of uncertainties and subjectivity. The definition of the degree of membership $\mu(x)$ of variables in a fuzzy set can be represented by different types of graphs. The most common types in the literature are triangular, trapezoidal, Gaussian, and sigmoidal (MARRO et al., 2010).

The triangular model is a simple representation, with a peak (maximum value) and a simple linear rise and fall. The trapezoidal model is like the triangular shape, but with a plateau at the top, useful for representing constant intervals. The Gaussian model is smooth and continuous, ideal for natural phenomena (SILVA et al., 2013). And the sigmoidal model shows gradual growth/decrease, ideal for trends (OLIVEIRA et al., 2014).

According to Fernandes (2005), choosing the most appropriate membership function is not a trivial task. However, in fuzzy systems, whose membership function parameters are completely defined by the expert, triangular and trapezoidal membership functions are more appropriate and straightforward.

The inference process evaluates the compatibility levels of the inputs with the antecedents of the various rules, activating the consequents with intensities proportional to them. This results in a fuzzy set, which will be converted to a scalar set in the defuzzification stage, providing the system's output (OLIVEIRA JÚNIOR, 1999, p. 65). In other words, fuzzy inference is the process of applying fuzzy rules to determine an output based on the inputs. There are several inference methods, but for this research, we chose the Mamdani method, widely used in fuzzy decision systems (HARB and AL-SMADI, 2006; MARRO et al., 2010; SILVA et al., 2019).

Linguistic variables described by membership functions are the basis of rules. To achieve this, the construction of a set of "If-Then" rules is necessary to combine the criteria provided by experts or extracted from numerical data. Mamdani's fuzzy rule is presented according to Equation 1 below (HARB and AL-SMADI, 2006):

Rule R_k: **IF** antecedent x_1 belongs to set A_{k1} **AND** antecedent x_2 belongs to set A_{k2} **THEN** consequent y belongs to set B_k (1)

For example, if distance (x1) belongs to the set short (Ak1) and last visit (x2) belongs to the set recent (Ak2), then priority (y) belongs to the set low (Bk1). These rules are applied to determine the responses. Therefore, at this stage, the degree to which a fuzzy rule is triggered is assessed by combining fuzzyified inputs. Various logical operators can be used depending on the research constraints for decision-making. Mamdani's minimum inference limits the activation degree of the rule (α_k) of the output to the lowest degree among the fuzzy inputs, thus considering a conservative method that ensures that the output is not stronger than the weakest input (Equation 2) (CORDÓN et al., 2001).

$$\alpha_k = min(\mu_{Ak1}(x_1), \mu_{Ak2}(x_2)) \quad (2)$$

Where,

- α_k: Activation degree of the rule;

- $\mu_{Ak1}\,(x_1)$: Membership degree of input x_1 to the fuzzy set A_{k1};
- $\mu_{Ak2}(x_2)$: Membership degree of input x_2 to the fuzzy set A_{k2};
- min: Logical operator (AND) that returns the smallest value between $\mu_{Ak1}\,(x_1)$ e $\mu_{Ak2}(x_2)$.

Then, to adjust the consequent, the membership degree of the fuzzy set associated with the consequent (Bk) is adjusted by activating the rule (α_k), as per Equation 3.

$$\mu_{adjusted,Bk}(y) = min(\alpha_k, \mu_{Bk}(y)) \tag{3}$$

Where,

- α_k: Rule activation degree;
- $\mu_{Bk}(y)$: Original membership function of the consequent set Bk;
- $\mu_{adjusted,Bk}(y)$: Adjusted function, considering the contribution limited by the activation degree.

If there is more than one input variable, it is necessary to apply an aggregation technique to the antecedent sets to generate a consequent set for each inference rule. The combination of these consequent sets will generate a final output set that is generally the result of the union (maximum) operator (SILVA et al., 2019). In other words, aggregation combines the activations of multiple rules, thus capturing the largest contribution among all rules to a single fuzzy output set, as described in Equation 4 (CORDÓN et al., 2001).

$$\mu_{aggregate} = \max\left(\mu_{adjusted,B1}(y), \mu_{adjusted,B2}(y), \dots, \mu_{adjusted,Bn}(y)\right) \tag{4}$$

Where,

- $\mu_{aggregate}$: Final membership function of the fuzzy output, resulting from the combination of the adjusted functions of all activated fuzzy consequents;
- $\mu_{adjusted,B1}(y), \mu_{adjusted,B2}(y), \dots, \mu_{adjusted,Bn}(y)$: Adjusted membership functions of the fuzzy consequent sets;
- max: Logical "OR" operator used to combine the adjusted functions of the fuzzy consequent sets.

The defuzzification step is characterized by converting a fuzzy set with degrees of membership into a single numerical value. The defuzzification method used in this research was the Centroid of Gravity (COG) method, which, according to Calvo (2006), is a widely used technique in fuzzy logic to convert a fuzzy set into a crisp (numeric) value. It is particularly effective because it considers the entire shape of the output fuzzy set, representing the weighted average of the membership function graph (MARRO et al., 2010). Equation 5 presents defuzzification in its discrete representation.

$$COG = \frac{\sum_{i=1}^{n} \mu_{aggregate}(y_i)\,.\,y_i}{\sum_{i=1}^{n} \mu_{aggregate}(y_i)} \tag{5}$$

Where,

- COG: Refers to the numerical value resulting from the defuzzification process, representing the equilibrium point of the fuzzy set in the output domain;
- y_i: Discrete points belonging to the domain of the output variable y;
- $\mu_{aggregate}(y_i)$: Final membership degree associated with each point y_i, obtained after aggregating the membership functions resulting from the activated fuzzy rules.

3. Methodology

The proposed fuzzy logic-based method, which aims to facilitate the diagnosis and planning of RAAP field activities, was implemented in stages using QGIS 3.34 (QGIS DEVELOPMENT TEAM, 2025) and the development of a Python routine using the Scikit-Fuzzy 0.5.0 library (WARNER et al., 2024).

3.1 Data preparation

The input data were extracted from the website of the National Spacial Data Infrastructure (Infraestrutura Nacional de Dados Espaciais – INDE) (INDE, 2025) using the INDE Viewer. Layers were added by theme, selecting the "Geodetic Networks" layer. From there, the shapefile for the "Altimetric Geodetic Network" theme was extracted. Figure 1 shows the access path.

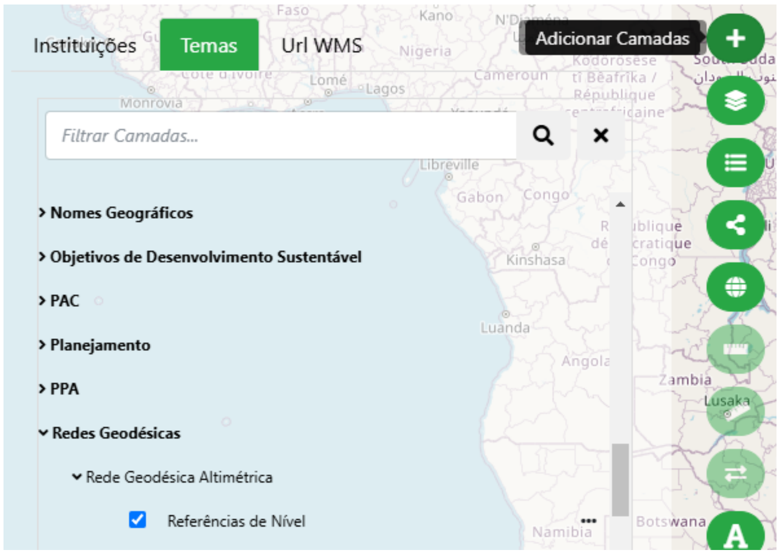

Figure 1 – Path to download the level references in the INDE Viewer.
Source: Authors (2025).

An additional theme was introduced for the IBGE's Geodesy and Cartography Management Offices (GGC), located in Santa Catarina, Rio de Janeiro, Goiás, the Federal District, Bahia, Ceará, and Pará, which served as the basis for determining the distances between the BMs (Figure 2). The location is available on the IBGE website (2025).

The characteristics of the BMs evaluated in this study were the distance between the BMs and the management units, the last visit, as well as qualitative status variables (destroyed, not found, good), and whether they were connected to GNSS stations, as it is understood that the priority is to obtain as many BMs as possible tracked by GNSS. To calculate the last visit in years, it was necessary to establish a time frame defined as February 5, 2025, thus allowing the conversion of visit dates into values expressed in years.

Since the objective of the study was to propose a proposal that would provide information for decision-making, the distance variable required a detailed analysis process. Calculating a simple linear distance would not yield a reliable diagnosis. To assist in this step, the OpenStreetMap (OSM) database (OPENSTREETMAP, 2024) available on the Geofabrik GmbH website (2024) was used to determine the routes between the management units and the BMs. The QGIS Network Analysis Toolbox 3 (QNEAT3) plugin (RAFFLER, 2025), designed for network analysis, was used to calculate shortest routes. Some routes, which passed through indigenous reservations or locations where the water-crossing technique was used, required vectorization. For example, the connection between Belém and Macapá, where vectorization of the maritime route was chosen, was used. Calculating distances using the OSM database required considerable computational effort, requiring partitioning into five blocks to cover all regions of the country. Once this preparation phase was completed, the next step was the calculation process.

Figure 2 – Geographical location of IBGE management offices.
Source: Authors (2025).

3.2 Aplication of fuzzy logic

The basic fuzzy logic process involved the following steps: fuzzification, inference, and defuzzification. This step used a routine developed in Python, with processing performed in Google Colab (GOOGLE, 2017).

3.2.1 Fuzzification

The input variables for analyzing the leveling network were benchmark (BM), management, distance, last visit, status, and GNSS connection. The last four variables were transformed into membership degrees using membership functions. The membership interval is [0, 1], with "0" representing an element that does not belong to a given set and "1" representing a full membership. Values between 0 and 1 represent partial membership degrees. This step presents a subjectivity inherent to fuzzy logic that will determine the following steps. Proper calibration by an expert is essential, as poorly sized fuzzy sets can result in inaccurate interpretations of linguistic variables, affecting the reliability and consistency of the results obtained (ÖZKAN and TÜRKŞEN, 2014). This can lead to inappropriate decisions or decisions that are incompatible with the reality of the problem being analyzed. For this research, the following representations were adopted to determine the membership functions (Table 1) and their graphical visualization (Figure 3):

Table 1 – Fuzzy set structure for the research's linguistic variables.

Linguistic variable	Membership function	Fuzzy set
Short distance (km)	Trapezoidal	[0,0,500,800]
Middle distance (km)	Trapezoidal	[600,1000,1500,2000]
Long distance (km)	Trapezoidal	[1800,2100,4500,4500]
Recent visit (years)	Trapezoidal	[0,0,8,12]
Moderate visit (years)	Triangular	[10,15,20]
Old visit (years)	Trapezoidal	[18,24,80,80]
BM Status (destroyed)	Triangular	[0,0,1]
BM Status (not found)	Triangular	[0.5,1,1.5]

BM Status (good)	Triangular	[1,2,2]
GNSS connection (yes)	Triangular	[0,0,1]
GNSS connection (no)	Triangular	[0,1,1]
Low Priority (%)	Trapezoidal	[0,0,20,50]
Medium Priority (%)	Trapezoidal	[30,40,60,70]
High Priority (%)	Trapezoidal	[50,80,100,100]

Source: Authors (2025).

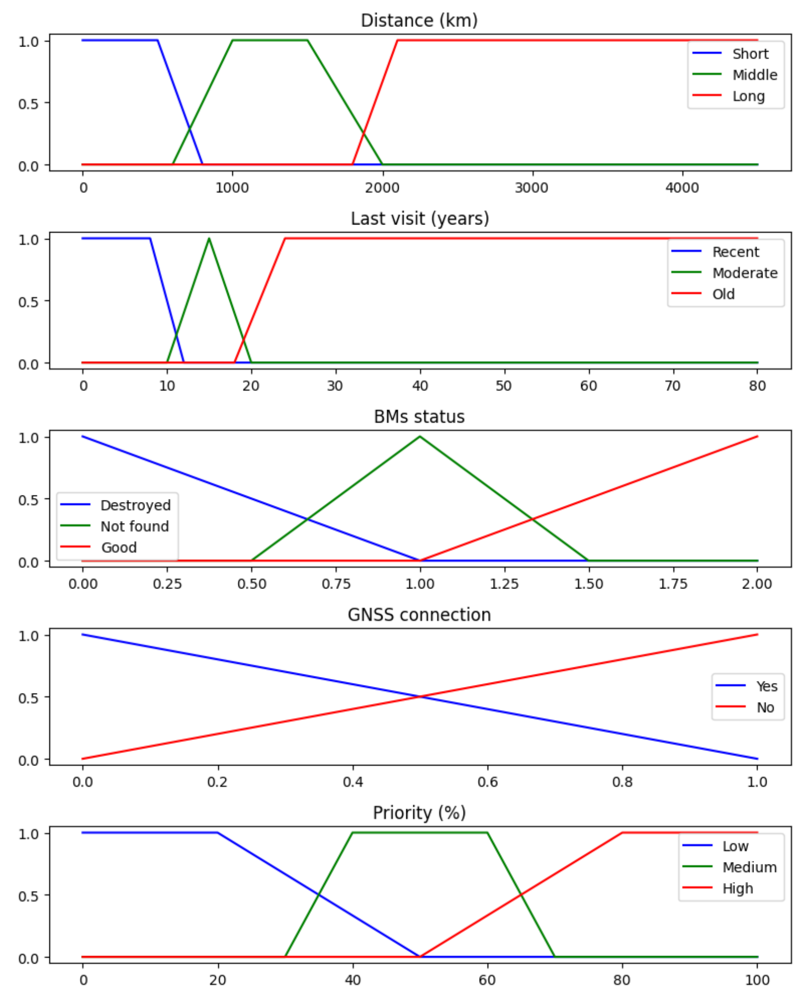

Figure 3 – Graphical representation of membership functions.
Source: Authors (2025).

In modeling the distance variable, short distances were defined as those less than 500 km in relation to the respective management, with a gradual decline in relevance up to 800 km. However, distances greater than 600 km could already be classified as medium, with this categorization being more consolidated in the range between 1,000 and 1,500 km. From 1,500 km onward, a progressive transition is observed, in which relevance to the average set gradually decreases until 2,000 km. The transition zone occurs at distances from 1,800 km onward, which are commonly interpreted as long, while those greater than 2,100 km are consistently classified as long.

The last visit variable was modeled to consider recent visits, those made up to 8 years ago, with a gradual transition up to 12 years. Moderate visits reach their maximum relevance in 15 years, with a smooth transition zone between 10 and 20 years. Visits over 24 years old are considered to be of maximum relevance, while there is a gradual transition starting at 18 years.

Regarding the RN status, the destroyed status is fully satisfied at point 0, with smooth transitions up to point 1. And the good RN status is satisfied at point 2, with smooth transitions starting at point 1. The most sensitive and abundant linguistic variable is the RN not found, which can transition between destroyed and good, since, in the absence of traces, the RN may have been truly destroyed or be in a difficult-to-access location. Expanding this registry with questions related to improving the quality of coordinates and information on engineering projects, such as road expansions, urban renovations, among others, can make this fuzzy set more robust.

Regarding the GNSS connection, interpretation is straightforward, as the goal is to obtain the largest possible number of RNNNs connected by GNSS. In this sense, a registry indicating the possibility of tracking can make this fuzzy set more representative of the field reality.

To facilitate the analysis of the defuzzification stage, the priority categories (low, medium, and high) were modeled in terms of relevance percentage, contributing to the categorization stage for spatial visualization.

3.2.2 Inference

A set of 24 rules was constructed to diagnose the RAAP. These rules were applied to determine the responses. Using the data from this survey as an example, if a BM has not been visited in over 40 years and its status in the database is good, the priority is high. Table 2 below presents each of the survey rules.

Table 2 – Set of rules adopted for inference.

Rules	IF	AND	THEN
1	short distance	recent visit	low priority
2	short distance	moderate visit	medium priority
3	short distance	old visit	high priority
4	middle distance	recent visit	low priority
5	middle distance	moderate visit	medium priority
6	middle distance	old visit	high priority
7	long distance	recent visit	low priority
8	long distance	moderate visit	medium priority
9	long distance	old visit	high priority
10	recent visit	destroyed status	low priority
11	recent visit	not found	low priority
12	recent visit	good status	low priority
13	moderate visit	destroyed status	low priority
14	moderate visit	not found	low priority
15	moderate visit	good status	medium priority
16	old visit	destroyed status	low priority
17	old visit	not found	low priority
18	old visit	good status	high priority
19	destroyed status	connected	low priority
20	destroyed status	not connected	low priority
21	not found	connected	low priority
22	not found	not connected	low priority
23	good status	connected	low priority
24	good status	not connected	high priority

Source: Authors (2025).

Using the rules defined according to Equation 1, the activation levels of the fuzzy rules are determined, as shown in Equation 2. With the activation levels defined, the consequents are adjusted, establishing the link between the rules and the corresponding output set, according to Equation 3.

The final inference step is aggregation, as per Equation 4, which consolidates the individual results of the fuzzy rules into a single fuzzy output set. Usually, the max operator is used to combine the activations of multiple rules, thus capturing the largest contribution among all rules to a single fuzzy output set. In other words, aggregation is the process of combining the results of multiple fuzzy rules. This ensures that all rules influence the final output, maintaining a complete representation.

3.2.3 Defuzzification

In this step, Equation 5 was used to convert a fuzzy set with degrees of membership into a single numerical value. The defuzzification method used in this research was the Centroid of Gravity (COG) method. Figure 4 shows an example of a graphical output with the priority value calculated as a percentage. In this example, the value of 65.14% represents an medium priority for verifying the physical reality of the BM.

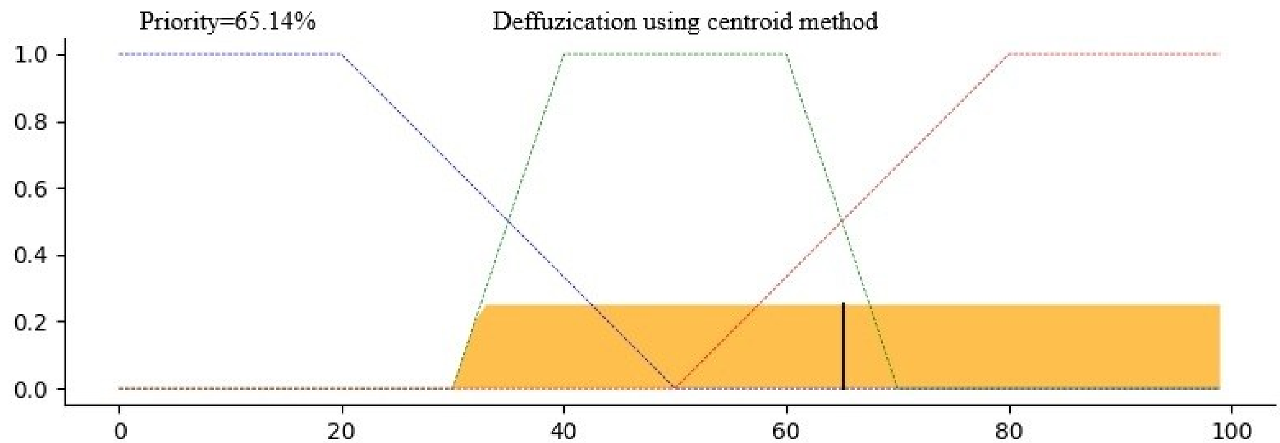

Figure 4 – Example of a graphical output resulting from defuzzification using the centroid method. Source: Authors (2025).

3.2.4 Sorting and Categorization

After defuzzification, the data were sorted based on the distance between the departments and the BMs to ensure hierarchical organization. This means that each BM has a list of departments that can act in order of priority. This helps with field activity planning. The data were then categorized into low, medium, and high priorities to facilitate decision-making (Equation 6).

$$Category = \begin{cases} Low, if\ priority\ \leq 50\% \\ Medium, if\ 50\% < priority \leq 70\% \\ High, if\ priority\ > 70\% \end{cases} \tag{6}$$

4. Results and discussion

The diagnosis of benchmarks 2025 priorities indicated high priority for 22,524 BMs, medium priority for 7,919 and low priority for 41,032 (Table 3).

Table 3 – Visitation priorities by management

Management	High priority	Medium priority	Low priority	Total
GGCBA	3,703	655	5,338	9,696
GGCCE	3,827	1,638	7,416	12,881
GGCDF	4,023	333	3,073	7,429

GGCGO	4,725	1,202	7,224	13,151
GGCPA	1,475	1,114	4,707	7,296
GGCSC	3,339	991	5,126	9,456
GLGC	1,432	1,986	8,148	11,566
Grand total	**22,524**	**7,919**	**41,032**	**71,475**

Source: Authors (2025).

Analyzing the high-priority results, it is striking that 376 BMs are located less than 50 km from a management unit, which initially highlights the need to resolve these low-cost cases with the utmost urgency. The range with the highest concentration of BMs requiring a physical reality check (PRC) is between 500 and 1,000 km. The Goiás management unit has BMs at considerably longer distances due to its responsibility for operating in the westernmost part of the Amazon region, where land access is more limited. Table 4 presents a summary by distance range. Figure 5 shows the high-priority BMs categorized by management unit.

Table 4 – Quantitative distribution of high-priority BMs by management unit and distance range.

Gerência	**X≤ 50 km**	**50 km <X≤ 100 km**	**100 km <X≤ 500 km**	**500 km <X≤ 1,000 km**	**1,000 km <X≤ 2,000 km**	**2,000 km <X≤ 3,000 km**	**3,000 km <X≤ 4,500 km**	**Total**
GGCBA	100	150	1,715	1,738	0	0	0	3,703
GGCCE	44	48	1,221	2,439	75	0	0	3,827
GGCDF	53	120	1,854	1,928	68	0	0	4,023
GGCGO	113	145	1,182	1,297	1,106	211	671	4,725
GGCPA	2	0	217	881	375	0	0	1,475
GGCSC	23	28	1,271	1,976	41	0	0	3,339
GLGC	41	30	781	580	0	0	0	1,432
Total geral	**376**	**521**	**8,241**	**10,839**	**1,665**	**211**	**671**	**22,524**

Source: Authors (2025).

Table 5 shows the stratification of high-priority by last visit. Of the 22,524 RRNNs, 19,388 have not been visited for over 20 years. The VRF of the RRNNs is essential to reversing this situation.

Table 5 – Last visit of high-priority BMs.

Last visit (years)	**Number of BMs**
70 < X	1,304
60 < X ≤ 70	1,077
50 < X ≤ 60	1,501
40 < X ≤ 50	5,219
30 < X ≤ 40	5,766
20 < X ≤ 30	4,521
10 < X ≤ 20	3,136

Source: Authors (2025).

Medium priorities consist of 7,919 BMs with a last visit recorded between 9 and 16 years ago and that do not have a GNSS connection (Figure 6). It should be noted that not all BMs are capable of GNSS tracking. However, this information is not available in the database used as a reference.

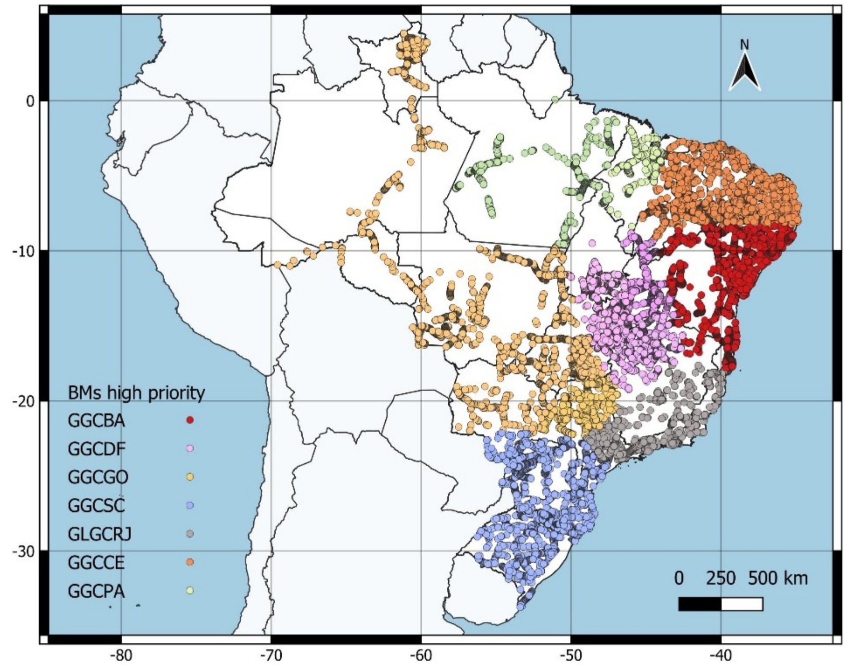

Figure 5 – BMs with high priority are categorized by management.
Source: Authors (2025).

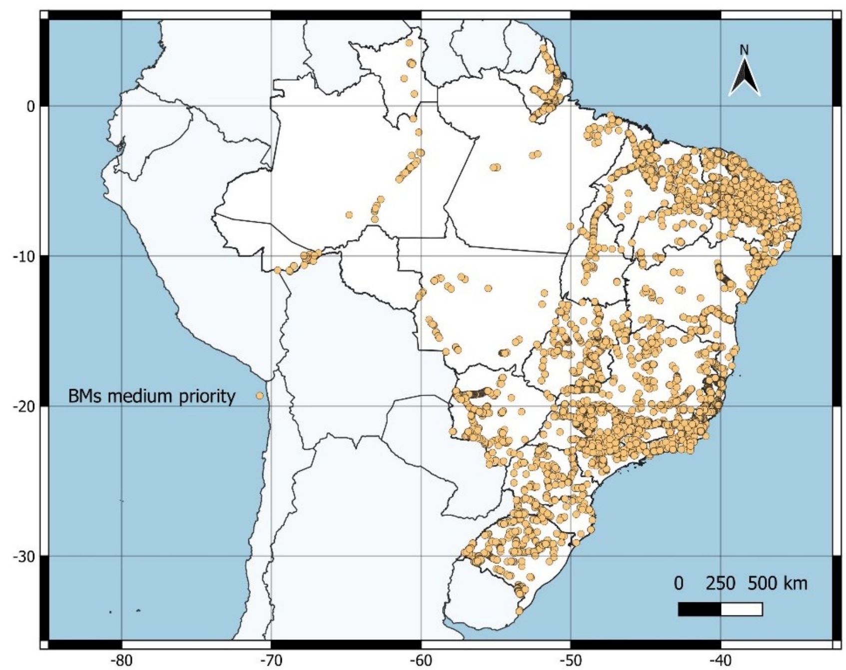

Figure 6 – BMs with medium priority.
Source: Authors (2025).

Low priority (Figure 7) is represented by BMs classified as good, not found, and destroyed. Those classified as good total 5,555, recently built or visited, of which 1,280 have GNSS connectivity. Destroyed BMs total 9,155, and not found, 26,322 (Table 6). Destroyed and not found represent 49.6% of the leveling network. This diagnosis may be even worse since of the 22,524 BMs classified as high priority and in good condition in the database, 19,388 have not been visited for more than 20 years.

Table 6 – Status of benchmarks with low priority.

Low priority	With GNSS connection	Without GNSS connection	Total
Good	1,280	4,275	5,555
Not found	55	26,267	26,322
Destroyed	62	9,093	9,155

Source: Authors (2025).

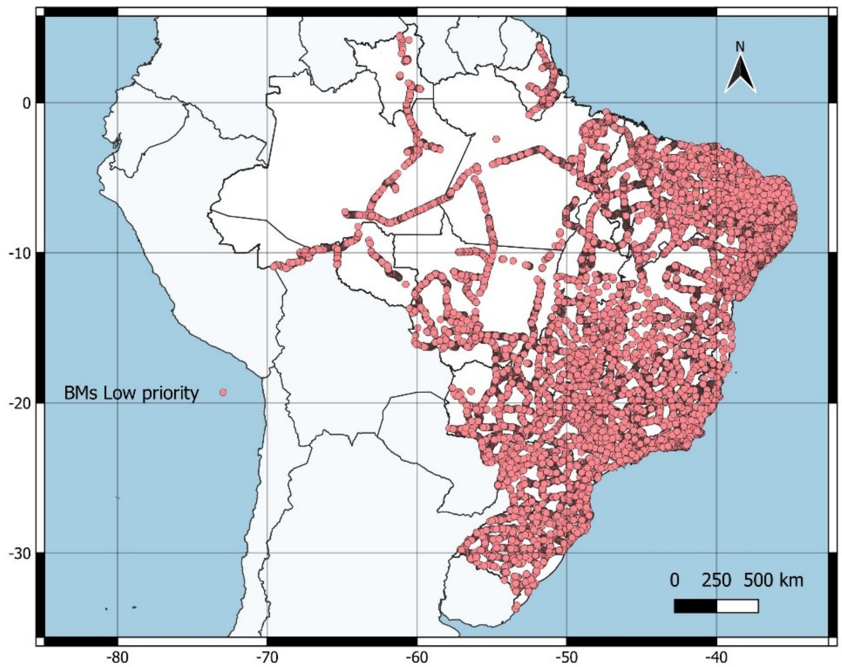

Figure 7 – BMs with low priority.
Source: Authors (2025).

5. Final considerations

A country's geodetic infrastructure, often an invisible component, allows for the minimization of country costs, enhancing the planning, development, and execution of activities with positional reliability and efficient integration between different sectors, such as transportation, environment, engineering, and land management, ensuring accurate support for decision-making and resource optimization.

The presentation of this methodological proposal, based on fuzzy logic, aimed to offer a robust and adaptable alternative to traditional methods, aligning with the complexity of decisions related to geodetic campaigns. Evaluating management priorities in the planning field activities offers several advantages, and the fuzzy system allowed for a more gradual representation of the analyzed variables, better reflecting human reasoning. Furthermore, fuzzy logic generates a clear priority output, which facilitates the ordering of benchmarks in a more refined manner, especially when allocating limited resources.

On the other hand, implementing this system requires specialized knowledge to correctly define the membership functions and fuzzy rules. This requirement becomes clear due to the subjectivity involved in modeling linguistic terms and selecting parameters, which can affect the consistency of results based on established rules. Therefore, it is recommended that a targeted survey be conducted with the experts responsible for planning the leveling network's field activities across all departments, with the goal of making the modeling of the parameters used in this research more robust and aligned with regional realities.

The main recommendation for using this methodology is to keep the database as up-to-date as possible. Furthermore, the situation and connection criteria can be supplemented in the future with more relevant information, such as whether there has been road duplication, whether the station is capable of GNSS connection, etc.

Determining routes based on OSM provided the diagnosis with the necessary feedback for planning field activities. More realistic distances will facilitate the input of campaign costs in the future, decisively aiding the specialist's analysis.

Regarding the RAAP diagnosis, the results highlight the urgent need to develop strategies for its densification and maintenance. Completely revitalizing the network in a country as large as Brazil is unfeasible, but strategic points in the network, such as future IHRF stations, network nodes, environmentally protected areas, and research of national interest using high-reliability benchmarks, can significantly contribute to its preservation and functionality. Furthermore, the height network has gained greater relevance in the space age, especially due to the need for ground reference points for accurate geodetic modeling.

Contemplating a participatory initiative with an annual award could expand knowledge and recognition of the importance of geodetic networks, contributing to the updating of the Geodetic Database, as many users currently have smartphones with cameras and GNSS.

Finally, the use of AI applied in this study represented a highly impactful innovation in the diagnosis and planning of the leveling network, aiding decision-making. Furthermore, recent advances, such as deep fuzzification integrated with machine learning (YANG; ZHANG; SHANGGUAN, 2025), point to the possibility of reducing expert dependence and improving the adaptability of fuzzy systems, which could be explored in future work on reference networks in Brazil.

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