PARACONSISTENT CONTRADICTION IN CONTEXT

Jonas Rafael Becker Arenhart *

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Resumo: Dizem que as lógicas paraconsistentes domesticam contradições: em tais lógicas, expressões como α e $\neg\alpha$ não trivializam a teoria. Neste sentido, podemos violar a Lei de Não-Contradição. O principal problema com esta caracterização diz respeito ao fato de que sempre que expressões como α e $\neg\alpha$ são ambas verdadeiras, *elas já não formam mais uma contradição*, mas no melhor dos casos uma *subcontrariedade*. Assim, talvez o maior desafio consista em explicar o que se quer dizer com uma 'contradição paraconsistente', ou seja, por um par de expressões como α e $\neg\alpha$ quando '$\neg$' é uma negação paraconsistente. Sugerimos que há um sentido razoável segundo o qual estas expressões podem ser entendidas, envolvendo a introdução de contextos distintos a partir dos quais α e $\neg\alpha$ são proferidas. Neste sentido, podemos ler estas expressões como formulando uma subcontrariedade and ainda atribuir um sentido intuitivo para 'contradições paraconsistentes'.

Palavras-chave: Lógica paraconsistente. Contradição. Subcontrária. Negação.

Abstract: Paraconsistent logics are said to domesticate contradictions: in such logics, expressions such as α and $\neg\alpha$ do not trivialize the theory. In this sense, we are able to violate the Law of Non-Contradiction. The main problem with this kind of characterization concerns the fact that whenever expressions such as α and $\neg\alpha$ are both true, *they are no longer a contradiction*, but at best a *subcontrariety*. So, perhaps the biggest challenge consists in explaining what is meant by a 'paraconsistent contradiction', that is, by a pair of expressions such as α and $\neg\alpha$ when '$\neg$' is a paraconsistent negation. We suggest that there is a sensible sense in which such expressions may be understood, involving the introduction of distinct contexts from which α and $\neg\alpha$ are uttered. In this sense, we can read those expressions as formalizing subcontrariety and provide for an intuitive meaning for 'paraconsistent contradictions'.

Keywords: Paraconsistent logics. Contradiction. Subcontrary. Negation.

*Professor of Philosophy at the Federal University of Santa Catarina, Florianópolis, Brazil.

Introduction

Paraconsistent logics were designed to deal with contradictions and contradictory statements. To confirm that the aim of paraconsistent logic is related with contradictions it is enough to check the many papers, books and collections published every year that intend to deal with paraconsistency. Once we employ a paraconsistent system of logic, we are told that we do not need to fear contradictions, given that the worst consequence of a contradiction — explosion — does not hold in such logics:

$$\alpha, \neg\alpha \nvdash \beta.$$

That is, in paraconsistent logics contradiction does not imply trivialization of the system (clear examples of such claims is to be found in authors having as different views on the subject as da Costa (COSTA, 1989), (COSTA N. C. A.; BUENO, 2006), and Priest (PRIEST, 2006)).

The most important consequence of such an analysis, it is said, is that one of the most famous laws of traditional logic, the *Law of Non-Contradiction*, is violated: we are allowed to have both α and $\neg\alpha$, true contradictions, as it were. We are also able to deal with contradictory objects, such as those featuring in Meinongian metaphysics, deal with inconsistent data bases, and, perhaps most importantly, keep rationality and sanity while still embracing the contradictions daily life and even our best scientific theories impose on us. The chapter of "Philosophical Motivations and Applications" of paraconsistent logics is indeed a long one.

Perhaps the greatest challenge to such a view of paraconsistency — and it seems to be the only view on paraconsistency available — is to make sense of so-called 'paraconsistent contradictions': what do the expressions α and $\neg\alpha$ mean exactly in paraconsistent logics? How can it be that both are true? Following Beall (BEALL, 2004, p.15), there are at least three distinct approaches to that question:

Weak paraconsistentist: The weak paraconsistentist believes that paraconsistent logics are just interesting mathematical models; useful, but not representing any real possibility. This is a kind of instrumental approach to paraconsistency.

Strong paraconsistentist: this kind of paraconsistentist believes that paraconsistent logics represent real possibilities, even though such possibilities do not obtain in the real world. That is, there are no true contradictions in fact.

Dialetheism: the Dialetheist paraconsistentist believes that some contradictions are true in this world, even if they are not true of concrete observable facts.

As we can see, the three kinds of paraconsistentists diverge on how deep a contradiction reaches. Anyway, and that is the point here, they all agree that paraconsistent logics deal with contradictions. However, there is still a problem of how should we understand this notion of contradiction. As Carnielli and Rodrigues (CARNIELLI; RODRIGUES, 2012, p.2) put it:

> It is a fact that paraconsistent logics permit extending conventional reasoning to make it able to deal with contradictions. However, the nature of the contradictions that are dealt with by paraconsistent logics is still an open issue.

It is curious to find out that contradictions could have distinct natures — in particular that their natures may depend on the systems of logic they appear in —, so that paraconsistent contradictions, in particular, have a nature not so well understood until now. Furthermore, having a nature distinct from classical contradictions, one wonders how paraconsistent contradictions can in fact be the violation of the classical contradiction, the one appearing in the classical law of Non-Contradiction. Now, leaving those oddities aside, our proposal in this paper is to address such an issue by somehow attempting to dissolve part of the problem (although not in a Wittgensteinian fashion). We shall advance the (already widespread) thesis that whenever paraconsistent logics are applied successfully — in philosophical analises of situations apparently involving a contradiction — we are in fact not dealing with a contradiction; in fact, we are dealing with subcontraries, a distinct (weaker) logical opposition. That claim may seem absurd at first, but we hope to make it reasonable by calling the reader's attention to the fact that whenever we violate the explosion law, by having a counter-model to the Explosion inference $\alpha, \neg\alpha \vdash \beta$, we must face the fact that the pair $\alpha, \neg\alpha$ represents only subcontraries.[1] So, in the end of the day, our proposal (we hope) seems to bring together the best of both worlds: we avoid the need to deal with contradictions (and to understand their nature in paraconsistent logics) and still make sure that applications of paraconsistent logic do reflect the phenomena addressed by them.

The plan of this paper is as follows. In section 2, to make the paper self-contained we review some of the terminology employed and present some arguments favoring a semantic treatment of the oppositions. In section 3 we briefly present some of the most well-known arguments to the fact that paraconsistent logics deal with subcontrariety, presenting the challenge to paraconsistent logics: making sense of paraconsistent contradictions, *i.e.*, expressions of the form α and $\neg\alpha$. In section 4 we present our proposal of informal interpretation of paraconsistent negation, which turn a paraconsistent contradiction into at best a relation of subcontrariety. We argue that this move somehow dissolves the problem of contradiction in paraconsistent logics. We conclude in section 5, and following Béziau (BÉZIAU, 2014) and (BÉZIAU, 2015), argue that we would be better not talking about contradictions in paraconsistent logics.

Contradiction and Other Oppositions

We shall now introduce some of the terminology that sets the stage for the problem we are dealing with. It is very common to define a contradiction in purely syntactical terms: two expressions are contradictory if they are of the form α and $\neg\alpha$, where $\neg$ is the negation sign of the language. There is, however, some good reasons for us not to follow this approach here. The first reason is that by accepting this definition of contradictory propositions we would be able to have an easy answer, an answer by definition, to the problem we are dealing with: any language with an operator that is supposed to play the role of a negation sign would automatically be able to represent contradictions. Now, there is good grounds to call such an attempt as begging the question, given that what we are investigating now is precisely the relation between negation and contradiction; for those pursuing this kind of answer, anyway, the problem of the 'nature of the contradiction' being represented would still be left hanging in the air.

Related with this first difficulty, there is another one that appears immediately after careful consideration of the previous paragraph: defining contradiction with the use of negation throws under the carpet too many important problems. For instance, it assumes that every sign that is supposed to act as a negation is really a negation, by the simple magical fact that we have stipulated it. As we know, that move goes just too fast; there is a great deal of philosophical discussion on which are the properties an operator *must* have to be legitimately called *negation*, and there is also just as much discussion about which properties a negation sign is allowed to

[1] As we shall see soon, that point is almost universally conceded by both friends and foes of paraconsistency; see Slater (SLATER, 1995) and Béziau (BÉZIAU, 2006).

lose while *still being a negation*. Considering an extreme case, Slater (SLATER, 1995), for one, does not consider paraconsistent negations as negations, so that assuming that we have a negation sign that always characterizes contradictions would amount to simply ignore the sceptic instead of arguing with him. In the end, conflating the concepts of negation and contradiction assumes to much to begin with (it takes those disputes on what negation is as irrelevant or as already solved). When there is so much at stake one cannot simply jump over such problems and assume by stipulation that a sign with a given number of properties is a negation.

There is a third main reason not to define contradiction syntactically, with the help of negation: traditionally, *contradiction is a semantic notion*. Given that our claim in this paper is that the syntactic expression in some cases does not capture the semantic content of a contradiction, defining contradiction in syntactic terms begs the question against any such discussion of 'expressive adequacy'. Also, there are cases in which we are clearly facing a contradiction but no negation is in front of one of the expressions. Consider the Aristotelian pair "Every men is mortal", as a contradictory of "Some men are not mortal". Of course, negation appears inside the second proposition, but it is not the case that we have here expressions of the form α and $\neg\alpha$, while it is at the same time clear that we are facing a contradiction, if anything is a contradiction. For another example, consider an electron that has just been measured for spin in a given direction. In this case, the propositions 'the spin is up' and 'the spin is down' are contradictory: both cannot be true (no electron can be measured both spin up and down at the same time), and both cannot be false, at least one of them is true (the measurement outcomes are comprised only between those two results). Again, we have no negation, but we still have a contradiction.[2]

But couldn't we adopt an inferentialist semantics and hold that the meaning of a logical constant such as negation is given by its introduction and elimination rules in a natural deduction system (under some reasonable conditions), for instance? We could, indeed. However, notice that distinct logical systems (classical, paraconsistent, intuitionist) have distinct rules for such connectives. That would imply that the meanings of negation are different in distinct systems, from which it follows that expressions of the form α and $\neg\alpha$ would also have distinct meanings. The best we could do in this case would be to speak of '*classical* contradiction', '*intuitionist* contradiction', '*paraconsistent* contradiction', and so on. That multiplication of contradictions would only contribute to the conclusion that when a paraconsistent logician says that the Law of Non-Contradiction does not hold, he is not in fact ..., well, contradicting, his fellow classical logician. To use an expression that appeared earlier, those formulas have distinct 'natures' in distinct systems. So, again, that is not the best option for our purposes.

It is well known that some paraconsistent logics were originally introduced as axiomatic formal systems; only later a semantics was developed. Anyway, even those systems were said to violate the Law of Non-Contradiction, mainly due to the fact that expressions such as $\alpha \wedge \neg\alpha$ were seen as not trivializing a theory in every case. The main point that we need to address is whether that expression is really a contradiction. To define contradictions semantically we employ the terminology of the square of opposition (following, for instance, Béziau (BÉZIAU, 2003)):

[2] Obviously, that same reasoning does not carry to the case of an electron in a superposition of the states 'spin up' and 'spin down' in a given direction, when no measurement is being carried out. For more discussions relating quantum mechanics and oppositions, in particular when superpositions are taken into account, see Arenhart and Krause (ARENHART; KRAUSE, 2016).

Contradiction: Propositions α and β are *contradictory* when α and β cannot both be true and cannot both be false.

Contrary: Propositions α and β are *contrary* when both cannot be true, but both can be false.

Subcontrariety: Propositions α and β are *subcontrary* when both can be true, but both cannot be false.

Again, notice that the definition is not in syntactical terms of a negation operator, but in semantic terms of truth values. Of course, definitions are judged not from the point of view of truth and falsity (*i.e.*, definitions are not true or false), but rather from their faithfulness, fruitfulness, utility and so on. We hope that the reader agree that the semantic definition of contradiction and of the other oppositions do satisfy those conditions.

It is in terms of those definitions of contradiction and subcontrariety that we shall explore the problem of understanding what exactly a pair of expressions of the kind α and $\neg\alpha$ means.

Paraconsistent Contradictions

Now, we can relate paraconsistent negations and the semantic opposition of subcontrariety in the following terms (following Béziau (BÉZIAU, 2006) and Dutilh Novaes (NOVAES, 2008)). Assume that we want to violate the Explosion rule

$$\alpha, \neg\alpha \vdash \beta.$$

What are we supposed to do? We must present a model (valuation, situation, world...) in which both α and $\neg\alpha$ are true, while β is false. However, when that is done and both α and $\neg\alpha$ are true we no longer have a contradiction. We have only the possibility of a subcontrariety. In fact, in general it is accepted that when α is false, then $\neg\alpha$ is true; this grants that we always have that at least one of the members of the pair is true (sometimes only one is true, sometimes both are true). This is enough to get a subcontrariety. In other systems, comprising paranormal logics, we have the possibility that both α and $\neg\alpha$ be false (see Béziau (BÉZIAU, 2015)). In these cases, however, we must also have both true when explosion fails. So, paraconsistent negations represent at best subcontrariety, but they do not represent contradictions.

Some problems now arise: how are we supposed to solve problems (philosophical, practical) involving contradictions by applying an operator that does not represent contradictions? Is that a legitimate move? Even though we shall not discuss those practical issues here, they are relevant for a proper understanding of the scope and limits of paraconsistency — and are related with our discussion to follow (for further discussions on these issues, see Arenhart (ARENHART, 2015)). Our concern in what follows will focus rather on what Dutilh Novaes (NOVAES, 2008) called "the challenge for the paraconsistentist", that is, explaining how can we distinguish situations in which it is legitimate to have both α and $\neg\alpha$ without explosion, from those cases in which α and $\neg\alpha$ cause explosion.

Challenge: How to distinguish cases in which contradictions are explosive — classical contradictions — from cases in which contradictions are not explosive — paraconsistent contradictions?

In other words, we shall spell the problem by employing the terminology we introduced in the previous section: how to distinguish cases which may be dealt with by a subcontrariety forming operator from those cases in which a real case of a contradiction is involved? As it is implicit in the challenge advanced by Dutilh Novaes, there are cases in which not even a paraconsistent logic is enough to avoid explosion.

Notice that this attitude towards contradictions is distinct from Priest's (PRIEST, 2004) claim that even though *some* contradictions may be true, it is not the case that *every* contradiction may be true. Priest's claim is based on the advice that sometimes, even though we could accept true contradictions, other considerations (such as simplicity and fertility of an alternative explanation to the phenomenon under consideration) may lead us to avoid such a path and look for a consistent treatment of the problem. Dutilh Novaes' claim is distinct: some contradictions could not be dealt with by a paraconsistent logic even if we wanted to. They are not amenable to treatment by a paraconsistent logic, or, as we shall say, they are not 'paraconsistent apt'. This is enough to express a deep disagreement on the nature of paraconsistent logic: in one case (Priest's), it seems implicit that a paraconsistent logic can be applied with no limitations; the only limit is imposed on us not from the nature of contradictions and paraconsistency, but from extraneous considerations such as the call of pragmatic virtues. We shall restrict ourselves to the problem raised by Dutilh Novaes. Here the problem is to distinguish where there are legitimate cases of application of paraconsistent logics: it is not possible to tame every contradiction with paraconsistent logics.

We believe that the proper answer to the question as to what exactly counts as a 'paraconsistent contradiction', to address the worries raised in the paper by Dutilh Novaes, may be found in the literature on paraconsistent logics. In fact, our strategy to address the issue will be to present and analyze here some of the cases in which paraconsistent logics are called to solve an apparent case of contradiction and to argue that there is no contradiction involved, but only a subcontrariety. What makes those cases 'paraconsistent apt' is precisely that: those are cases in which we may find a subcontrariety where an apparent contradiction is present. Cases that are 'paraconsistent inapt', that is, cases where contradictions lead to trivialization (*i.e.*, where there is really a contradiction, and not a subcontrary) are resistent to such an interpretation.

How do apparent contradictions become subcontrariety? As we shall see, statements that are usually taken as contradictories may be seen as subcontraries by the introduction of a kind of context before a statement and also before its negation. Taking the context into account transforms contradictions into subcontrarieties, and so we have a non-explosive situation.

But now the question may be: what is a context? As we shall see by the examples presented soon, they may be propositional attitudes, quantifiers, modal operators, points of views, and much more. Their aim is to transform a pair of contradictory sentences into a pair of subcontrary utterances.

Contradictions in Context

In this section we address directly the problem raised by the challenge presented in the previous section.

The first example we shall discuss comes from Jean-Yves Béziau, which in his recent paper (BÉZIAU, 2014) presents arguments to the effect that paraconsistent logics deal with subcontrariety, and not with contradictories. In fact, he was one of the first paraconsistent logicians to go on and advance the suggestion that we should stop speaking about contradictions, keeping only at best subcontrariety. By discussing this case here we are not implying that Béziau is not aware that paraconsistent logics do not deal with real cases of contradiction; rather the other way around (although Béziau still speaks about 'contradictory viewpoints'). This example is being discussed because we think of it as paradigmatic case of application of paraconsistency, illustrating the strategy of introducing contexts we mentioned before.

What is more relevant for us is his case study of a paraconsistent system which incorporates the idea of contradictory points of view, and this illustrates perfectly the kind of strategy of introduction of a context to render a contradiction into a subcontrariety. Béziau considers an aspect of the quantum mechanical description of reality: the paradoxical wave-particle duality. As it is well-known, in the context of the famous two-slit experiment, for example, quantum objects behave sometimes as waves, sometimes as particles.[3] In this kind of experiment, a coherent light source is sent through a plate having two parallel slits; there is a screen after the slits that registers the arrival of the items[4] going through the slits. When the light beam is sent and both slits are open, the wave nature of light makes the light that passed through both slits interfere, so that a pattern is generated on the screen after the plate; a pattern, indeed, that would be expected of the behavior of a wave (see the picture in (GHIRARDI, 2005, p.50), for instance). On the other hand, when only one slit is open, the pattern created by the particles on the screen after the slit is just what would be expected from particles going through the single slit, no interference in this case.

Bohr and the so-called *Copenhagen Interpretation* of Quantum Mechanics interpreted the situation through the famous *Complementarity Principle*: particles behave both as waves and as particles, but never in the same context.[5] It is the experimental apparatus that determines what kind of observation will result, but we never experiment both aspects; they are complementary and provide for a complete — even if both are excluding — description of reality.

From a logical point of view, the idea of distinct points of views can be easily formalized, as Béziau (BÉZIAU, 2014) has shown, by employing the tools of modal logic and possible world semantics (which are then rebaptized as viewpoints) with a universal accessibility relation. Let us represent by P the statement that O is a particle and by Q the statement that O is a wave. Consider a point of view w_1, according to which O behaves as a particle; then, according to w_1, it is true that O is a particle ($v(P,w_1) = V$), and false that it is a wave ($v(Q,w_1) = F$). Now, consider another point of view w_2, according to which O is a wave. Then, according to w_2 it is true that O is a wave ($v(q,w_2) = V$, and it is false that O is a particle ($v(P,w_2) = F$). The situation may be put schematically thus:

Viewed from w_1**:** $v(P,w_1) = V$, $v(Q,w_1) = F$.

Viewed from w_2**:** $v(Q,w_2) = V$, $v(P,w_2) = F$.

Assuming an universal accessibility relation between worlds, Béziau defines a negation as follows:

$$v(\sim p, w_i) = F \text{ in point of view } w_i \text{ if } v(p,w) = V \text{ for every point of view } w.$$

That is, the negation of a proposition p is false in a point of view w_i if p is true in every point of view associated with w_i. Let us put that definition to work in the example of the particle

[3] We shall not present all the details here; for further discussions and details, see for instance Ghirardi (GHIRARDI, 2005).

[4] We use the term 'items' here because it seems neutral as to whether those items are particles or waves or objects of any kinds. Given that the problem surrounds precisely the status of those entities, a neutral word seems advisable.

[5] Again, there is much controversy about the precise formulation of the Principle and its intended meaning, but we refer the reader to (GHIRARDI, 2005) for further details and references.

O. In our example, $v(P, w_1) = V$. Given that there is an associated point of view w_2 such that P is false in that point of view ($v(P, w_2) = F$), we have that $v(\sim P, w_1) = V$. That is, both P and $\sim P$ are true at w_1. However, that does not mean that we have a contradiction: this is a case of subcontrariety; in fact, '$\sim$' is a paraconsistent negation. Furthermore, we do not have something O being both a wave and a particle. It is the peculiar way that the paraconsistent negation is introduced that makes the points of view disappear, and put in the same level, let us say, P and $\sim P$, which, on its turn, leads us to think that we are facing a contradiction.

Having said that, let us discuss how it illustrates the general thesis that paraconsistency involves the introduction of a context. From the brief presentation we have described before, it is clear that in the language for the system described, with the semantics of point of view, we can have both P and $\sim P$ true. However, the intuitive reading of this situation does not allow us to read it as a contradiction; we do not read P and $\sim P$ as stating that 'O is a particle' and 'O is not a particle'. As the application to particles illustrates, what we really have when the paraconsistent logic applies is distinct points of view from which a proposition is judged. In fact, P is true means that *from a given point of view* w_i the proposition is true. $\sim P$ is true in w_i means that *from a distinct point of view* the proposition P is false. This is hardly surprising, given how the construction that was made. However, as we shall argue, it is this kind of introduction of a hidden context — in this case a 'point of view' — that makes sense of paraconsistent contradictions.

This is also the same strategy behind the construction of discussive logics (see the details in (COSTA N. C. A.; BUENO, 2006, pp.844-49)). By beginning with a model of $S5$ and allowing that a proposition is true in discussive logic whenever it is true in one world of the model, we are allowing that, taken collectively, propositions p and $\neg p$ may be true. In fact, they are never true *at the same world*, but it only happens that p is true in one world $_i$, while $\neg p$ is true at another world w_j. Given that worlds represent participants of a discussion group, the pair p and $\neg p$ may both be true, that is, it is a case of a subcontrariety again. Also, its intuitive interpretation introduces the hidden context: "participant w_i states…", and "participant w_j states ….

What is common to both examples and to others we shall present soon is the fact that from the point of view of the paraconsistent language in which propositions are regimented, we have a proposition p and its negation $\neg p$. However, as we have discussed in the previous section (and see also (NOVAES, 2008)), a contradiction needs not be characterized by negation. It is in fact a semantic notion. Now, the fact that we are in face of a subcontrariety appears only when we shift back to informal language, when a context that disappears in the formal language needs to be reintroduced to get the informal meaning of the propositions.

The same principle may be found in discussions of application of paraconsistent logics to other group situations, as for instance a disagreement of a group of referees (see (CARNIELLI; RODRIGUES, 2012)) or the construction of expert systems or knowledge bases of a given domain D. In the last case (the reasoning applies also to the case of referees), we may (and probably will) find experts disagreeing about particular cases. Consider the case of doctors evaluating the symptoms of a given patient (this example is taken from (COSTA N. C. A.; BUENO, 2006, p.865)):

> … given the same observable symptoms, doctor d_1 may believe that the patient has a virus infection, doctor d_2 may conclude that the patient has an allergic reaction, while doctor d_3 may say that the patient either has a viral infection or an allergy, but not both. It is clear that if we had used the opinions of these three doctors in our knowledge base, we would be led into an inconsistency.

In this case, the data base would simply "ignore" the fact that the opinions come from distinct sources. That is, what we really have is, for instance, that "According to doctor d_1, the patient has a virus infection", and, let us say, "According to doctor d_2, the patient has no virus infection". Again, as far as that is all that we have, this is a case of a subcontrariety, not a case of contradiction. The patient, by herself, does not 'have and not have' a virus infection (or allergic reaction). So, the intuitive meaning of the propositions bring them back to the context in which they are subcontraries. That is, from the syntactical form p and $\neg p$ in the formalization of the judgements constituting the data base we may forget that we are dealing with subcontrariety. However, the intuitive meaning re-introduces the contexts and keeps sanity. The contexts, in this case, are:

"According to doctor d_1 the patient has a virus infection."

"According to doctor d_2 the patient has not a virus infection."

Another case corroborating the general thesis of this section concerns partial truth and its applications (see da Costa and French (COSTA N. C. A., 2003), da Costa, Krause and Bueno (COSTA N. C. A.; BUENO, 2006)). Let us summarize the general idea. Consider a domain of knowledge Δ and our scientific theorizing about it. We begin by forming a set D consisting of the observable entities in Δ, enlarged later with unobservable entities to account for observable behavior. Given a particular binary relation R holding between the elements of D, we do not always know whether $\langle d_i, d_j\rangle \in R$ or $\langle d_i, d_j\rangle \notin R$. In actual science, it may reasonable be expected that we don't know which is the case. Partial relations account for that and make for a more realistic description of science. Besides the pairs $\langle d_i, d_j\rangle$ which are in R and those which are not, partial relations are characterized by the ordered pairs of which we are ignorant about whether they do belong to R or not. A partial structure is a structure $\mathfrak{A} = \langle D, R_i\rangle_{i\in I}$, where R_i is a family of partial relations. Now, sometimes we can extend a partial structure to a total structure by putting, for each relation of the structure, the ordered pairs $\langle d_i, d_j\rangle$ of whose statuses we are ignorant either in R or definitively out of R. This constitutes, as we mentioned, an extension of a partial structure $\mathfrak{A}$ into a total structure $\mathfrak{B}$. We can now evaluate for the (Tarskian) truth of propositions of the language in $\mathfrak{B}$, given that it is a Tarskian structure. We say that a sentence α is partially true in $\mathfrak{A}$ when there is a total structure $\mathfrak{B}$ in which α is true in the Tarski style.[6]

Now, it is easy to see that both α and $\neg\alpha$ may be partially true in some cases: suffices to have a partial structure $\mathfrak{A}$ such that there are two extensions of it, one $\mathfrak{B}$ in which α is true, and another, $\mathfrak{C}$ in which $\neg\alpha$ is true. So, a contradiction may be partially true, at least? Is that a contradiction? Not really. As the reader may have noticed, α and $\neg\alpha$ are not true *in the same structures*, but rather in total structures $\mathfrak{B}$ and $\mathfrak{C}$, respectively, which extend $\mathfrak{A}$. There is again the introduction of a context that makes a subcontrariety appear, and when the intuitive meaning is sought, we find that distinct contexts make a proposition and its negation partially true.

We could go on and present further examples, but we believe that those are enough to illustrate the general thesis (see also the discussions in Arenhart (ARENHART; KRAUSE, 2016)). When a paraconsistent logic is called to deal with a contradiction, what we are really facing is a subcontrariety. The syntactical form of α and $\neg\alpha$ (when '$\neg$' is a paraconsistent negation) leads us to think that we are facing a contradiction, but we should recall that a contradiction is not defined in terms of negation and that there are the hidden contexts playing their role.

This provides an answer to the challenge present by Dutilh Novaes we mentioned before. Recall that the challenge consists in specifying when we may employ a paraconsistent logic

[6] There are much more details and technicalities in the definition of partial truth; however, for our purposes such a brief account suffices. The reader may check (COSTA N. C. A., 2003) also (COSTA N. C. A.; BUENO, 2006, sec.4.3).

to deal with a contradiction and when the contradiction is just too strong to be dealt with by employing such logics. The answer that arises from the above considerations puts many facts already established in the literature together to tailor an answer to the question. Let us check how it works.

Recall that a paraconsistent negation, in order to be paraconsistent, will have to be at most a subcontrary forming operator. That is, as Béziau (BÉZIAU, 2006, p.23) has put it: when a negation sign forms contradictions it is not paraconsistent, and when it is paraconsistent, it does not form contradictions. So, when we speak about 'paraconsistent contradictions' we are speaking about cases of subcontrarieties (at best). Paraconsistent logics are applicable whenever we have a situation in which an apparent contradiction may be seen as a subcontrariety by the appeal to the appropriate contexts. That is, we must find what are the (almost always) hidden contexts that make the conflicting propositions 'paraconsistent able', *i.e.*, able to be such that a proposition and its negation are both true. As we have seen, such contexts may include modal operators, points of view, propositional attitudes and much more. When such contexts are available we are authorized to apply paraconsistent logics. In a nutshell, *when the contradiction is not really a contradiction, but rather acts as a subcontrariety, there paraconsistent logics are applicable*. The formal apparatus of such logics are best suited to deal with such situations.

As a final problem, let us briefly the question of whether there could be a limit to this kind of "context insertion" strategy. Does every contradiction allow for the introduction of a hidden context, so that paraconsistency applies to every apparent contradiction? It does not seem so. Let us consider the reasons. In a recent debate, Beall (BEALL, 2011) and Eldridge-Smith (ELDRIDGE-SMITH, 2012) illustrate such a situation in which it seems doubtful whether one can reasonably introduce a context. Consider Pinocchio stating the sentence "My nose will grow". According to the story, Pinocchio's nose grows if and only if he tells a lie. So, we are facing a paradox, and in the end Pinocchio's nose will grow and not grow. Beall (BEALL, 2011) does not think that this is a case of a real contradiction. What is really at stake, according to him, is that we have hidden contexts in those statements: "According to the story, Pinocchio's nose will grow", and "According to the story, Pinocchio's nose will not grow". There is not a real contradiction.

So far, so good; this "in the story" introduction illustrates the use of contexts to generate situations in which paraconsistent logics apply, where contradictions become subcontraries. However, Eldridge-Smith (ELDRIDGE-SMITH, 2012) reasonably asks for what makes the Pinocchio case liable to such a treatment inside contexts, while by the lights of some paraconsistentists there are other cases, such as those involving the Liar Paradox, that are not liable to such treatments. That is, as Beall seems to believe, the Liar Paradox seems to involve a real case contradiction, but the Pinocchio case seems to be rendered innocuous by the introduction of contexts. What is the difference? As Eldridge-Smith (ELDRIDGE-SMITH, 2012) puts, the difference seems to rely on metaphysical stipulation, which on their turn must be argued for, not merely stipulated. This bears directly on the question of how to distinguish between paraconsistent apt 'contradictions' and paraconsistent inapt contradictions. The question then is: how could one provide for a principled reasoning to distinguish such cases where a contradiction legitimately appears from those in which a context may be introduced to avoid contradictions?[7]

There may be no principled way to distinguish between the relevant contexts, but an example may help us illustrate what is going on. As we mentioned throughout the section, paraconsistent negation seems apt when there is a contexts inside which it is introduced. The problem arises

[7] Notice that Beall and Eldridge-Smith do not put the problem in those terms, however.

when the negation can somehow escape the context, that is, the real contradictions appear when from a pair Cp and $C\neg p$ we pass on to a pair Cp and $\neg Cp$, where C represents a context and p is a proposition. Let us follow the usual terminology and call "Exclusion" the property a context have when from $C\neg p$ we can legitimately infer $\neg Cp$. It seems exclusion is relevant to answer the question about when a context is able to allow a paraconsistent contradiction.

Now, as we have mentioned, this is not to be taken as a categorical answer, but rather as a first step in this direction. The issue may be clearly illustrated by a discussion advanced by da Costa and French (COSTA N. C. A., 2003, pp.98-100). The main problem they are considering here is whether belief in a contradiction (defined syntactically) would entail contradictory beliefs; that is, by using **B** to be the belief operator, the question is whether $\mathbf{B}(p \wedge \neg p)$ entails somehow $\mathbf{B}p \wedge \neg\mathbf{B}p$. In the first case, notice, the contradiction is confined to the scope of the operator, it is an "internal contradiction", while in the second case, the contradiction spreads outside of the operator, it is an "external contradiction", and we have contradictory beliefs.

The crucial step in going from internal contradictions to external ones (that is, reaching out of the context) concerns application of the Exclusion property:

1. $\mathbf{B}(p \wedge \neg p)$ (assumption)
2. $\mathbf{B}p \wedge \mathbf{B}\neg p$ (distribution of **B**)
3. $\mathbf{B}p \wedge \neg\mathbf{B}p$ (from 2, with Exclusion)

So, the main problem, at least in the case we are considering, seems to grant that we may prevent that the belief operator indeed obey the Exclusion property. According to the diagnosis by da Costa and French (COSTA N. C. A., 2003, p.99), the main problem comes from the association of "belief" with "believing that is true". By assuming that "belief that p" ($\mathbf{B}p$) is synonymous with "belief that p is true" ($\mathbf{BT}p$), the authors claim that we obtain the Exclusive property for belief (see again da Costa and French (COSTA N. C. A., 2003, p.99)):

1. $\mathbf{BT}\neg p \rightarrow \mathbf{B}\neg\mathbf{T}p$ (exclusion for truth)
2. $\mathbf{B}\neg\mathbf{T}p \rightarrow \neg\mathbf{BT}p$
3. $\mathbf{B}\neg p \rightarrow \neg\mathbf{B}p$ (dropping **T** from 2)

Obviously, the problem here concerns step 2. If that is an instance of Exclusion for belief, then the claim by da Costa and French according to which belief results exclusive because of its association with truth is ungrounded. However, they seem to think that that step is justified mainly because of properties of truth:

> [...] the exclusive nature of beliefs seems to follow from that of truth, together with the standard understanding of "belief that p". We might say that there is nothing inherent in belief itself that causes it to be exclusive; it's all in the propositions and whether they are regarded as true or not. (da Costa and French (COSTA N. C. A., 2003, p.99))

Now, even though that conclusion is not obvious from the derivation offered (step 2 seems either to require exclusion for beliefs or else to be mysterious), we may follow the general suggestion that what prevents a paraconsistent context involving belief from becoming a classical context is the fact that one avoids that belief is exclusive, that is, that a negation may become external to a context. Shifting to weaker notions of truth, such as quasi-truth, may allow one to block such inferences and keep belief without exclusion. In this sense, a scientist, for instance, may belief in a contradiction (*i.e.* $\mathbf{B}(p \wedge \neg p)$, the contradiction is inside a context) without having contradictory beliefs (*i.e.* $\mathbf{B}p \wedge \neg\mathbf{B}p$, the contradiction is outside the context).

So, when we reason about our beliefs, $\neg(\mathbf{B}p \wedge \neg\mathbf{B}p)$ should hold, while an appropriate doxastic logic should also allow for $\mathbf{B}(p \wedge \neg p)$. As da Costa and French put it: "when we reason about our own beliefs, our "external" logic is classical, whereas our "internal" logic is paraconsistent" (da Costa and French (COSTA N. C. A., 2003, p.100)). That is, when contradictions are confined to the context of belief, we reason with paraconsistent logics, allowing for contradictions inside the context. When the contradiction spreads out, we are in the field of classical logic, and that case must be avoided.

This illustrates rather well our suggestion that paraconsistent negations are always hiding a context, a context which allows the propositions involved to become subcontraries. When we get out of the context and the propositions become contradictions, then, it is classical logic that is operating.

Final Remarks

If the arguments presented in the previous sections are correct, there is a way in which we can employ tools already available in the literature in order to address the challenge to the paraconsistentist: 'paraconsistent contradictions' are cases of subcontrariety, which may be seen as not really contradictory when we evaluate the statements by taking into account the contexts of utterances. That idea, as we mentioned earlier, helps us making sense of paraconsistent applications while keeping it consistent with well-known facts about paraconsistent negations (it generates subcontraries, and not contradictories).

What the suggestion does not solve is the problem of contradictions. In fact, contradictions are still to be avoided, they do trivialize a system. So, the usual contention that paraconsistent logics help us violate the Law of Non-Contradiction will have to be revised. The Law of Non-Contradiction appears to be violated in paraconsistent logics due to syntactical similarities of formulas in classical and in paraconsistent logics. However, a 'paraconsistent contradiction' does not seem to be a contradiction in the sense that Aristotle condemned. Those are rather distinct things.

Bearing that in mind, some of the *Philosophical Applications* of paraconsistent logics will also have to be revised and a humble attitude to be assumed. It is said that paraconsistent logics deal with the realm of contradictory objects of Meinongian ontology, that it allows us to have Russell's set understood, and that it allows us to be rational in the presence of contradictions. A more realistic attitude towards such applications seems to be advised. As Béziau (BÉZIAU, 2014), (BÉZIAU, 2015) remarked, it would be closer to the facts if we stopped speaking about contradictions when dealing with paraconsistent logics and kept only subcontrariety. That is much less spectacular, but is much closer to the truth.

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